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AMERICAN MATHEMATICAL SOCIETY

# *Notices*

*Edited by* J. H. CURTISS

*VOLUME 5, NUMBER 1* *ISSUE NO. 29* *FEBRUARY 1958*

## *Contents*

MEETINGS

Calendar of Meetings . . . . . 2
Program of the February Meeting in New York . . . . . 3

PRELIMINARY ANNOUNCEMENTS OF MEETINGS . . . . . 6

NEWS ITEMS AND ANNOUNCEMENTS . . . . . 9

PERSONAL ITEMS . . . . . 15

NEW PUBLICATIONS . . . . . 18

MEMORANDUM TO MEMBERS . . . . . 19

ABSTRACTS OF CONTRIBUTED PAPERS . . . . . 20

If you attend a meeting announced in this issue of the NOTICES, please bring your copy to the meeting. There will be only a limited number of copies at the registration desk.

Please send in abstracts of papers to be presented in person well in advance of the deadline.

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# MEETINGS

## CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

| Meeting No. | Date | Place | Deadline for Abstracts |
|---|---|---|---|
| 544 | April 18-19, 1958 | Chicago, Illinois | Mar. 5 |
| 545 | April 18-19, 1958 | Stanford, California | Mar. 5 |
| 546 | April 24-26, 1958 | New York, New York | Mar. 5 |
| 547 | June 20, 1958 | Corvallis, Oregon | May 7 |
| 548 | August 25-30, 1958 (63rd Summer Meeting) | Cambridge, Massachusetts | July 11 |
| 549 | October 25, 1958 | Princeton, New Jersey | Sept. 11 |
| | November, 1958 | Evanston, Illinois | |
| | January 20-22, 1959 (65th Annual Meeting) | Philadelphia, Pennsylvania | |
| | Summer Meeting, 1959 | Salt Lake City, Utah | |
| | November, 1959 | Detroit, Michigan | |

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## FIVE HUNDRED FORTY-THIRD MEETING

New York, New York
February 22, 1958

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## PROGRAM

The five hundred forty-third meeting of the American Mathematical Society will be held at Hunter College in New York City on Saturday, February 22, 1958.

Professor Louis Nirenberg of New York University will deliver an address entitled On partial differential equations of elliptic type in Room 300 at 2:00 P.M. by invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings.

Sessions for contributed papers will be held at 10:30 A.M. and at 3:15 P.M. in Room 300. (The preliminary announcement stated that the meeting would begin at 10:00 A.M., but the later time has been set because of the small number of contributed papers.)

Hunter College is on Park Avenue between 68th and 69th Streets. It can be reached most readily by means of the Lexington Avenue subway. Guests arriving on Saturday are asked to use the 69th Street entrance. Room 300 is on the third floor at the Park Avenue end of the building. A registration desk will be found near this room.

In accordance with a recently adopted policy of the Society, abstracts of contributed papers will no longer be published in the BULLETIN, but will appear in the NOTICES instead. Abstracts of papers presented at some recent meetings appear elsewhere in these NOTICES. It is intended that, beginning with this issue of the NOTICES, abstracts of papers which are to be presented in person at meetings will appear in the same issue of the NOTICES as the program of the meeting. There will be cross references to the abstracts in the program.

Thus, for example, the title of paper (1) in the program below is followed by (543-5), indicating that the abstract can be found under the designation 543-5 among the published abstracts. All of the in person contributed papers for this meeting of the Society appear on pages 94-99 of these NOTICES.

## PROGRAM OF THE SESSIONS

(Time limit for each contributed paper, 10 minutes)

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### SATURDAY, 10:30 A.M.

General Session, Room 300

(1) Integral representation of semi-groups of unbounded self-adjoint operators (543-5)
Professor A. E. Nussbaum, Rensselaer Polytechnic Institute

(2) Similarity and unitary equivalence for normal operators (543-6)
Mr. A. N. Feldzamen, Yale University

(3) Maximum principle for bounded functions (543-9)
Professor John Wermer, Brown University

(4) Duality in general ergodic theory (543-10)
Dr. Robert Heyneman, Cornell University

(5) A characterization of Hermitian manifolds (543-8)
Dr. Wilhelm Klingenberg, Institute for Advanced Study

(6) On maximum principles for nonhyperbolic partial differential operators (543-13)
Dr. Philip Hartman and Mr. Richard Sacksteder, Johns Hopkins University

### SATURDAY, 2:00 P.M.

General Session, Room 300

On partial differential equations of elliptic type (One hour)
Professor Louis Nirenberg, New York University

### SATURDAY, 3:15 P.M.

General Session, Room 300

(7) A special class of Boolean algebras (543-4)
Professor L. J. Heider, Institute for Advanced Study

(8) On a class of topologies for division rings. Preliminary report (543-11)
Dr. Ellen Correl, University of Maryland

(9) Ideals of index 1 in group rings (543-2)
Mr. G. O. Losey, Princeton University

(10) A circular parity switch and applications to number theory (543-7)
Dr. Daniel Shanks, David Taylor Model Basin

(11) The non-existence of certain algorithms of finite automata theory. Preliminary report (543-12)
Professor J. R. Buchi, Mr. C. C. Elgot and Mr. J. B. Wright, University of Michigan

SUPPLEMENTARY PROGRAM
(To be presented by title)

(12) A note on monotonely decreasing functions
Professor J. W. Andruskiw, Seton Hall University
(13) Multiplicative norms for metric rings. I
Mr. Silvio Aurora, Jackson Heights, New York
(14) Multiplicative norms for metric rings. II
Mr. Silvio Aurora, Jackson Heights, New York
(15) Multiplicative norms for metric rings. III
Mr. Silvio Aurora, Jackson Heights, New York
(16) On power multiplicative norms. I
Mr. Silvio Aurora, Jackson Heights, New York
(17) On power multiplicative norms. II
Mr. Silvio Aurora, Jackson Heights, New York
(18) On certain regular metric rings
Mr. Silvio Aurora, Jackson Heights, New York
(19) Representations of even functions (mod r). III. Three notes
Professor Eckford Cohen, University of Tennessee
(20) Twisted polynomial hyperalgebras
Dr. Edward Halpern, University of Michigan
(21) On a question by Kaplansky in ring theory
Professor I. N. Hernstein, Cornell University
(22) Expansion of solutions of second order linear elliptic equations about infinity
Dr. N. G. Meyers, University of Minnesota
(23) Determinants of certain composite matrices
Dr. W. E. Roth, Austin, Texas
(24) The matric equation $x^2 - y^2 = A$
Dr. W. E. Roth, Austin, Texas
(25) On the structure of maximum modulus algebras
Professor Walter Rudin, University of Rochester
(26) The maximum achievable length of an error correcting code
Professor Jacob Wolfowitz, Cornell University and Israel Institute of Technology

R. D. Schafer
Associate Secretary

Storrs, Connecticut
January 9, 1958

## PRELIMINARY ANNOUNCEMENTS OF MEETINGS

### FIVE HUNDRED FORTY-FOURTH MEETING

CHICAGO, ILLINOIS
April 18-19, 1958

The five hundred forty-fourth meeting of the American Mathematical Society will be held at the University of Chicago, Chicago, Illinois, on Friday and Saturday, April 18-19, 1958. All sessions will be held in Eckhart Hall.

Registration will be in the Common Room on the second floor of Eckhart Hall, beginning at 9:00 A.M., Friday.

Sessions for the presentation of contributed papers will be held at 10:00 A.M. on Friday and Saturday, and 3:15 P.M. on Friday.

There will be a special session on Saturday for the presentation of papers which failed to meet the deadline. Further details will be available at the registration desk.

The facilities of Hutchinson Commons, a dining hall directly across from Eckhart Hall, will be available to members of the Society and guests for all meals.

The following hotels have agreed to accommodate those members of the Society making reservations in advance:

| | Single | Double |
|---|---|---|
| **In the University district** | | |
| Shoreland Hotel<br>5454 South Shore Drive | $7.00 up | $9.00 up |
| Del Prado Hotel<br>5307 South Hyde Park Blvd. | 7.00-11.00 | 9.00-13.00 |
| Hotels Windermere<br>1642 East 56th Street | 6.50-8.50 | 8.50-11.00 |
| Hotel Broadview<br>5400 South Hyde Park Blvd. | 4.00-6.00 | 6.00-8.00 |
| Hotel Miramar<br>6218 South Woodlawn | 3.50-5.50 | 5.00-7.00 |
| Hyde Park Y.M.C.A.<br>1400 East 53rd Street | 2.25 | |
| **In the Loop district** | | |
| The Conrad Hilton | 6.50-10.00 | 12.00 up |

The prices listed above are subject to change.

Reservations should be made directly with the hotel.

Complete details will be found in the final program in the next issue of the NOTICES.

J. W. T. Youngs

Associate Secretary

Bloomington, Indiana
January 9, 1958

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## FIVE HUNDRED FORTY-FIFTH MEETING

### STANFORD, CALIFORNIA
April 18-19, 1958

The five hundred forty-fifth meeting of the American Mathematical Society will be held on Friday and Saturday, April 18-19, 1958, at Stanford University, California.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be an address at 10:00 A.M. Saturday by Professor W. W. Rogosinski of the University of Colorado and the University of Durham. His topic is General moment problems.

By invitation of the same Committee, there will be a Symposium on Friday afternoon and evening, devoted to Banach Algebras and Harmonic Analysis. The Program Committee for the Symposium consists of Professors Edwin Hewitt, Chairman, I. I. Hirschmann, Henry Helson, and H. A. Dye.

Sessions for contributed papers will begin at 1:00 P.M. on Saturday, and will be followed by a tea at the Bowman Alumni House.

There are numerous excellent motels within easy driving distance of the University. (A detailed list may be obtained from the Chamber of Commerce, 725 University Avenue, Palo Alto, California.) For those who travel to the meeting by means of public transportation, the following hotels are recommended: President Hotel, Palo Alto; Cardinal Hotel, Palo Alto. Bus service is available between these hotels and the University.

V. L. Klee, Jr.

Associate Secretary

Seattle, Washington
January 10, 1958

## FIVE HUNDRED FORTY-SIXTH MEETING

New York, New York
April 24-26, 1958

The five hundred forty-sixth meeting of the American Mathematical Society will be held on Thursday, Friday, and Saturday, April 24-26, 1958, at Columbia University.

A Symposium on Combinatorial Designs and Analysis (sponsored by the Society with the aid of the Office of Ordnance Research) will be held on Thursday and Friday. There will be four sessions of the Symposium: I. Existence and construction of combinatorial designs: II. Combinatorial analysis of extremal problems: III. Problems of communications, transportation and logistics: IV. Numerical analysis of discrete problems. All sessions of the Symposium will be held in the McMillin Theatre. Further information will appear in the next issue of these NOTICES, or may be obtained from the chairman of the Program Committee for the Symposium, Professor Marshall Hall, Jr., of Ohio State University.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor I. M. Singer of the Massachusetts Institute of Technology will address the Society on Friday at 2:00 P.M. on Connections and holonomy groups, and Professor J. C. Moore of Princeton University will deliver an hour address entitled A survey of some modern developments in homotopy theory on Saturday at 2:00 P.M. Both hour addresses will be given in the Pupin Physical Laboratories, Room 301.

There will be sessions for contributed papers at 3:15 P.M. on Friday, and at 10:00 A.M. and 3:15 P.M. on Saturday.

The Employment Register will be maintained on Friday afternoon and on Saturday.

The Council of the Society will meet at 5:00 P.M. on Friday.

Columbia University may be reached by the Broadway - 7th Avenue line of the IRT Subway at the 116th Street station. The McMillin Theatre is on the north side of 116th Street at Broadway. The Pupin Physical Laboratories are on the south side of 120th Street at Broadway. A registration desk will be found near the regular sessions of the meeting at the times of the sessions.

Further details of the meeting will appear in the next issue of the NOTICES. Abstracts of contributed papers should be sent to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, so as to arrive PRIOR TO THE DEADLINE, March 5.

R. D. Schafer
Associate Secretary

Storrs, Connecticut
January 9, 1958

## NEWS ITEMS AND ANNOUNCEMENTS

NATIONAL SCIENCE FOUNDATION TO SUPPORT AFTER-SCHOOL-HOURS TRAINING FOR HIGH SCHOOL SCIENCE AND MATHEMATICS TEACHERS. The National Science Foundation announced today that it will accept proposals from universities and colleges interested in sponsoring In-Service Institutes for Secondary School Teachers of Science and Mathematics to be held during the academic year 1958-59.

A primary purpose of the In-Service Institute program is to assist colleges and universities in their efforts to encourage teachers in outlying school districts to take advantage of scientific training facilities not otherwise readily accessible to them. The Foundation has already announced support of 125 Summer and Academic-Year Institutes in colleges and universities throughout the Nation during the summer of 1958 and the following school year.

In-Service Institutes for secondary school teachers of science and mathematics will offer work in the subject matter of science and mathematics especially designed for these teachers. Institute meetings will be held outside regularly scheduled school hours -- e.g., evenings, Saturdays, or late afternoons -- so that teachers may attend while still teaching full time in their schools. A typical Institute might meet once a week for two hours for the full academic year of about 30 weeks. Half of these meetings might, for example, be devoted to laboratory work.

It is possible for a particular institution to offer courses in more than one subject-matter area in these in-service programs. Courses could be offered simultaneously, though not necessarily on the same evening, in mathematics, physics, chemistry, biology, etc., for instance. Any given participant should probably be limited to attending no more than two of these if he is to find time to do a good job on each. The In-Service Institute program contemplates that each group will be kept to less than thirty members so that discussion may be full and free.

Foundation support to some twenty-five In-Service Institutes will cover all tuition and fees, plus any other direct costs to the college or university directly attributable to the program. Though the Foundation does not provide stipend support for participants in the in-service program, the NSF grants provide funds to underwrite travel expenses in connection with attendance at the Institutes. Deadline for submission of completed proposals to the Foundation is March 15, 1958. Directions for preparing proposals may be obtained from the Division of Scientific Personnel and Education, National Science Foundation, Washington 25, D. C.

A SYMPOSIUM ON COMBINATORIAL DESIGNS AND ANALYSIS (sponsored by the Society with the aid of the Office of Ordnance Research) will be held on Thursday and Friday, April 24-25, at Columbia University in conjunction with the meeting of the Society in New York, New York on April 25-26. Some details of the Symposium appear in the preliminary announcement of that meeting on page 8 of these NOTICES. A complete program of the Symposium will appear in the next issue of the NOTICES.

A SYMPOSIUM ON NUMERICAL APPROXIMATION sponsored by the Mathematics Research Center, U. S. Army, will be held April 20 to 23 at the University of Wisconsin, Madison. The topics of the symposium include linear approximation, interpolation, Tchebycheff and other extremal approximations, expansions and algorithms.

One-hour surveys (including a survey of recent Russian literature) and 30-minute research papers will be presented. Time for discussion is provided between lectures. The following speakers from abroad have agreed to participate: L. Collatz, L. Fox, Z. Kopal, C. P. Miller, A. Ostrowski and E. L. Stiefel. It is intended to have the proceedings of the symposium published.

All persons interested in attending the symposium are urged to write to Professor R. E. Langer, Director, Mathematics Research Center, U. S. Army, University of Wisconsin, 1118 W. Johnson Street, Madison 6, Wisconsin.

THIRD U. S. NATIONAL CONGRESS OF APPLIED MECHANICS will be held at Brown University, June 1-14, 1958. The Congress program, which is to be distributed in May of 1958, will feature over one hundred papers in Applied Mechanics, including mechanics of rigid bodies, mechanics of deformable solids, mechanics of fluids and gases, thermodynamics, and heat transfer. In addition, there will be four general lectures by leading authorities.

To reduce the cost of attending the symposium to the utmost, the following all-inclusive living arrangement is offered. This covers rooms in the new West Quadrangle of Brown University, and all meals from Breakfast on Wednesday, June 11, through Luncheon on Saturday, June 14, but NOT the Banquet on Thursday night. The rates will be $24.00 for each of two persons sharing a room, or $30.00 for a person occupying a single room. Rooms will be available without additional charge for Tuesday night and Saturday night. Please note that this arrangement was only made possible by excluding all deviating arrangements for those planning to live on the Brown campus. Those planning to stay at hotels or motels will be able to lunch in the Brown Refectory at the price of $1.50 per luncheon. The banquet ticket will be $5.00. The registration fees are as follows: Advance Registration on or before June 7th, $4.00; Registration on June 8th

or later, $6.00. Please make checks payable to: Applied Mechanics Congress, not to any person.

Requests for copies of the program and letters concerning advance registration should be addressed to Miss E. M. Addison, Box F, Brown University, Providence 12, Rhode Island. In registering, please indicate number of persons in party and family relationship, desire for single rooms or willingness to share rooms, estimated arrival and departure times, and need for parking space.

A COMPUTING CENTER AT SOUTHERN METHODIST UNIVERSITY. Southern Methodist University announces the opening of a Computing Laboratory on its campus. A new building houses the Univac Scientific 1103 Computer, the Remington Rand Service Bureau and the SMU Computing Laboratory offices and classrooms. The computer is operated jointly by Remington Rand as a service to industry and by SMU as an academic service for research and teaching. The SMU operation is associated with the University's New Graduate Research Center. Professors and students have free use of the machine for academic research and training in computer work. Training programs are available for faculty and students. Computing projects are now underway in the fields of engineering, mathematics, psychology, law, religion, management and others. SMU will make the computer available to other universities and nonprofit institutions on a cooperative arrangement involving only a nominal fee for overhead, and invites inquiries leading to such use of the machine. SMU regards its laboratory as a regional university computing facility.

NEW MEMOIRS. MEMOIR No. 28, by Ernst Snapper, is now available. It is entitled "COHOMOLOGY GROUPS AND GENERA OF HIGHER-DIMENSIONAL FIELDS". The list price of this MEMOIR is $1.90 with the usual 25% discount to members of the Society. "The purpose of this Memoir is to give and investigate birationally invariant definitions of genera and allied notions of algebraic varieties. Let then $k \subset E$ be two arbitrary fields, where E is obtained from k by field-adjunction of a finite number of elements. Each projective model of E/k is considered as a sheaf whose stalks are the local rings of the points of the model. These sheaves form a "transitive system of sheaves" whose "limit sheaf" R has as base space the Riemann manifold of E/k and whose stalks are the algebraic valuation rings of E/k. We define the q-th cohomology group $H^q(E/k)$ of E/k as the q-th cohomology group of R. Denoting the k-dimension of the k-vectorspace $H^q(E/k)$ by $h^q(E/k)$, we define $\sum_{q \geq 0}(-1)^q h^q(E/k)$ as the arithmetic genus, $h^1(E/k)$ as the irregularity, and $h^r(E/k)$ as the geometric genus of E/k; here, r is the degree of transcendency of E/k. Of course, the arithmetic genus of an irreducible algebraic variety is now simply defined as the arithmetic genus of its field of

rational functions, and the same for the other notions. The main investigation of the numerical invariants $h^q(E/k)$ centers around their relationship with the differentials of E/k, their finiteness, and their behavior under a place of E/k. Part I of the Memoir develops the notions of the limit, the cohomology groups and the exact sequences of a transitive system of sheaves. This Part I is purely topological in nature and can be read without knowledge of algebraic geometry."

PROFESSOR RICHARD COURANT was honored by New York University at a convocation on Professor Courant's 70th birthday, which was Wednesday, January 8. Dr. John E. Ivey, Executive Vice President of New York University, delivered the citation honoring Professor Courant. At the convocation, the establishment of the Richard Courant Lectureship in Mathematical Sciences at New York University was announced. This new biennial series of public lectures was established primarily by contributions from students, colleagues, and friends of Professor Courant. The lectures will commence in the academic year 1958-59.

CORRECTION. The following item appeared on page 30 of the October 1957 issue of the NOTICES: Professor W. T. Reid of Northwestern University has been appointed to a professorship at the University of California, Los Angeles. It should have been made clear that Professor Reid is on leave from Northwestern University and that he has been appointed to a visiting professorship at the University of California, Los Angeles.

BACKLOGS OF CERTAIN MATHEMATICAL RESEARCH JOURNALS. Information on this important matter is being published twice a year in the NOTICES, with the kind cooperation of the respective editorial boards.

It is important that the reader should interpret the data with full allowance for the wide and sometimes meaningless fluctuations which are characteristic of them. The waiting times in particular are affected by many transient effects, which arise in part from the refereeing system. Extreme waiting times as observed from the published dates of receipt of manuscripts may be very misleading, and for that reason, no data on extremes are presented in the table below.

| | 1 | 2 | 3 | | 4 | 5 | | |
|---|---|---|---|---|---|---|---|---|
| Journal | No. Issues per year | Approx.No. pages published currently per year | Backlog 11/30/57 Pages | Backlog 5/31/57 Pages | Estimated Current Waiting Time Months | Observed Waiting time in latest issue $Q_1$ Mos. | Med. Mos. | $Q_3$ Mos. |
| Transactions | 6 | 1,650 | 1,300 | 1,160 | 18 | 15 | 16 | 19 |
| Proceedings | 6 | 1,000 | 100 | 170 | 9 | 8 | 9 | 12 |
| American Jnl. | 4 | 950 | NR[a] | NR[a] | NR[a] | 4 | 7 | 8 |
| Annals | 6 | 1,200 | 300 | 350 | 12 | 10 | 12 | 14 |
| Duke Journal | 4 | 700 | 160 | 200 | 12 | 7 | 9 | 10 |
| Canadian Jnl. | 4 | 600 | 90 | 106 | 8 | 11[b] | 14[b] | 17[b] |
| Annals of Math. Statistics | 4 | 1,250 | 100 | 180 | 9 | 8 | 11 | 14 |
| Quarterly of App.Math. | 4 | 450 | 250 | NR[a] | 15 | 14 | 15 | 16 |
| Michigan Math. Journal | 3 | 300 | 40 | NR[a] | 10 | 6[c] | 7[c] | 10[c] |
| Illinois Jnl. | 4 | 600 | 188 | NR[a] | 12 | 11 | 12 | 13 |
| Jnl. of Math. and Mech. | 6 | 1,000 | 0 | NR[a] | 6 | --[d] | --[d] | --[d] |
| SIAM Journal | 4 | 300 | 0 | NR[a] | 6 | 2 | 4 | 5 |
| Pacific Jnl. | 4 | 900 | 0 | 100 | 6 | 11[e] | 12[e] | 13[e] |

Footnotes: [a] NR means that either no response was received to a request for information, or else that the response contained data in another form.

[b] Issues are undated; these data based on the third number of the 1957 volume, which was arbitrarily counted as the September, 1957 issue.

[c] Issues are undated; these data based on the second number of the 1957 volume, which was arbitrarily counted as the September, 1957 issue.

[d] Dates of receipt of manuscripts not indicated in this journal.

[e] Issues are undated; these data based on "Summer" 1957 issue which was arbitrarily counted as the June, 1957 issue.

Some of the columns in the table above are not quite self-explanatory, and here are some further details concerning how the figures were computed.

Column 2. These figures are rounded off to the nearest 50.

Column 3. For each journal, this is the estimate as of the indicated dates, of the total number of printed pages which will have been accepted by the next

time that manuscripts are to be sent to the printer, but which nevertheless will not be sent to the printer at that time. (It should be noted that pages received but not yet accepted are being ignored.)

Column 4. Estimated by the editors (or the Editorial Department of the American Mathematical Society in the case of the Society's journals), and based on these factors: manuscripts accepted, manuscripts received and under consideration, manuscripts in galley, and rate of publication.

Column 5. The first quartile ($Q_1$) and the third quartile ($Q_3$) are presented, to give a measure of the dispersion which will not be too much distorted by meaningless extreme values. The median (Med.) is used as the measure of location. The observations were made from the latest issue received in the Headquarters Offices before the deadline date for this issue of the NOTICES. The waiting times were measured by counting the months from receipt of manuscript in final revised form, to nominal month of issue (not counting in month of receipt but counting in month of issue). It should be noted that when a paper is revised, the waiting time between receipt by editors of the final revision and its publication may be much shorter than is the case for a paper which is not revised, so these figures are to that extent distorted on the low side.

## PERSONAL ITEMS

(This section is restricted to members of the Society)

Assistant Professor Harley Flanders of the University of California, Berkeley, is on leave and has been awarded a National Science Foundation Fellowship at Cambridge University.

Associate Professor A. R. Harvey of San Diego State College has received a Fulbright award to lecture at the College of Arts and Sciences in Baghdad, Iraq.

Dr. Leonard Maximon of Fysisk Institutt, Norway, has been awarded a Turner and Newall Fellowship in theoretical physics at The University of Manchester for the year 1957-1958.

The National Science Foundation has announced grants in science to the following institutions, to support studies by the professors indicated: Brown University, W. S. Massey, D. A. Buchsbaum; University of California, Berkeley, David Blackwell, E. L. Lehmann, Michel Loève, Jerzy Neyman, Henry Scheffe, J. G. van der Corput; University of California, Los Angeles, E. A. Coddington; Catholic University of America, Eugene Lukacs; Columbia University, E. R. Lorch; University of Georgia, M. L. Curtis; Institute for Advanced Study, Frank Harary; University of Kansas, Robert Schatten; Kenyon College, Otton Nikodym; Louisiana State University and Agricultural and Mechanical College, R. D. Anderson; University of New Mexico, I. I. Kolodner; Purdue Research Foundation, A. H. Copeland, Jr.; University of Southern California, Herbert Busemann, R. S. Phillips, H. A. Dye; Syracuse University, P. G. Bergmann; Tulane University of Louisiana, A. D. Wallace, A. H. Clifford; Washington University, Harvey Cohn; University of Wisconsin, R. H. Bing.

Mr. H. H. Brown of the Ramo-Wooldridge Corporation has accepted a position as research mathematician for the Missile Systems Division of the Lockheed Aircraft Corporation, Palo Alto, California.

Mr. H. P. Carter of David Lipscomb College has accepted a position as senior mathematician at the Union Carbide Nuclear Company, Oak Ridge, Tennessee.

Dr. J. D. Esary of the University of California, Berkeley, has accepted a position as research engineer for the Boeing Airplane Company, Seattle, Washington.

Professor István Fáry of the Université de Montréal has been appointed to a visiting associate professorship at the University of California, Berkeley.

Mr. Lawrence Fearnley of the University of Utah has been appointed to an assistant professorship at Brigham Young University.

Mr. George F. Feeman of Muhlenberg College is on leave and is a Danforth Teacher at the Danforth Foundation, Saint Louis, Missouri.

Dr. Lawrence Goldman of Columbia University has been appointed to an assistant professorship at Stevens Institute of Technology.

Mr. Max Goldstein of the Los Alamos Scientific Laboratory has accepted a position as senior research scientist at the AEC Computing Center, New York University.

Mr. Arthur Kaufman of Queens College has been appointed to an assistant professorship at Staten Island Community College.

Professor G. M. Koethe of the University of Mainz has been appointed to a professorship at the University of Heidelberg.

Dr. R. S. Lehman, Jr. of the University of Göttingen has been appointed to an assistant professorship at the University of California, Berkeley.

Mr. Fred J. Lorenzen, Jr. of the University of New Hampshire has been appointed to an assistant professorship at Union College, Schenectady, New York.

Mr. G. E. Mahoney of SPICA, Inc., has accepted a position as senior technical staff member of the Airbourne Systems Laboratory of Radio Corporation of America, Waltham, Massachusetts.

Dr. Lawrence Markus of Princeton University has been appointed to an assistant professorship at the University of Minnesota.

Dr. M. A. Medick of City College has accepted a position as senior staff scientist at the Research Advance Development Division of AVCO Manufacturing Corporation, Lawrence, Massachusetts.

Dr. J. B. O'Toole of Hughes Aircraft Company has accepted a position as senior engineer for National Cash Register Company, Hawthorne, California.

Assistant Professor E. C. Paige, Jr. of the University of Illinois has been appointed to an assistant professorship at the University of Virginia.

Dr. W. O. Portmann of the National Advisory Committee for Aeronautics has been appointed to an assistant professorship at Case Institute of Technology.

Dr. W. F. Reynolds of Massachusetts Institute of Technology has been appointed to an assistant professorship at Tufts University.

Dr. D. W. Sasser of Yale University has accepted a position as staff member of Sandia Corporation, Albuquerque, New Mexico.

Dr. C. B. Shaw, Jr., of Lockheed Aircraft Corporation has accepted a position as research physicist for Hughes Aircraft Company, Culver City, California.

Mr. Marvin Shinbrot of Ames Aeronautical Laboratory has accepted a position as research scientist for Lockheed Aircraft Corporation, Palo Alto, California.

Assistant Professor G. L. Spencer, II, of the University of Maryland has been appointed to an associate professorship at Williams College.

Associate Professor Henry Van Engen of Iowa State Teachers College has been appointed to a professorship at the University of Wisconsin.

Dr. Jack Warga of Electro Data Corporation has accepted a position as senior staff scientist in the Research and Advance Development Division of Avco Manufacturing Corporation, Lawrence, Massachusetts.

Mr. N. T. Watson of Amherst, Massachusetts has been appointed to an assistant professorship at Clark University.

Dr. Morris Weisfeld of the Ramo-Wooldridge Corporation has accepted a position as mathematician for Shell Development Company.

Dr. J. W. Woll, Jr. of Princeton University has been appointed to an assistant professorship at Lehigh University.

The following promotions are announced:

J. J. Andrews, St. Louis University, to an associate professorship.

M. V. Johns, Jr., Stanford University, to an assistant professorship.

P. B. Johnson, Occidental College, to a professorship.

V. L. Klee, Jr., University of Washington, to a professorship.

C. J. Lewis, Fordham University, to an assistant professorship.

S. L. Ross, University of New Hampshire, to an assistant professorship.

E. W. Swokowski, Marquette University, to an assistant professorship.

Patrick Suppes, Stanford University, to an associate professorship.

The following appointments to instructorships are announced:

University of Chicago: Mr. W. S. Martindale, III; University of Minnesota: Dr. N. G. Meyers; Ohio State University: Mr. Morton Brown; University of Rochester: Mr. P. J. Cohen, Dr. A. B. Hajian; University of Washington: Dr. August Newlander, Jr; Yale University: Dr. Walter Koppelman.

## NEW PUBLICATIONS

Alblas, J. B. Theorie van de dreidimensionale spanningstoestand in een doorboorde plaat. [Theory of three-dimensional stress systems in plates with holes.] Amsterdam, Paris, 1957. 12 + 127 pp.

Apté, A. S. Recherches théoriques et expérimentales sur les mouvements des liquides pesants avec surface libre. (Publ. Sci. Tech. Ministère de l'Air, no. 333.) Paris, 1957. 5 + 115 pp. 1450 fr.

Borden, A. Time constants and frequence response of coated hot wires used as turbulance-sensing elements. (Hydromechanics Laboratory, Report 952.) David Taylor Model Basin, 1957. 8 + 48 pp.

Brillouet, G. Etude de quelques problèmes sur les ondes liquides de gravité. (Publ. Sci. Tech. Ministère de l'Air, no. 329.) Paris, 1957. 9 + 145 pp. 1700 fr.

Chu, C. M., Clark, G. C., and Churchill, S. W. Tables of angular distribution coefficients for light-scattering by spheres. Ann Arbor, University of Michigan Press, 1957. 15 + 58 pp. $3.00.

Churchill, S. W. See Chu, C. M.

Clark, G. C. See Chu, C. M.

Flammer, C. Spheroidal wave functions. Stanford University Press, 1957. 9 + 220 pp. $8.50.

Flammer, C. Variational formulae for domain functionals in electromagnetic theory. (Air Force Cambridge Research Center, Tech. Rep. 57.) Menlo Park, Stanford Research Institute, 1957. 3 + 23 pp.

Gaudfernau, C. L. Photoélasticité tridimensionnelle. Aspects théoriques et expérimentaux. (Publ. Sci. Tech. Ministère de l'Air, no. 330.) Paris, 1957. 5 + 86 pp. 1200 fr.

Papandreou, A. G. A test of a stochastic theory of choice. (University of California Publications in Economics, vol. 16, no. 1.) Berkeley-Los Angeles, University of California Press, 1957. 18 pp. $0.50

Reynolds, G. E. Table of (sin x)/x. Bedford, Electronics Research Directorate, Air Force Cambridge Research Center, 1957. 3 + 205 pp.

Steketee, J. A. An introduction to the equations of magnetogasdynamics. (Institute of Aerophysics, Review no. 9.) University of Toronto, 1957. 6 + 35 pp.

Zubov, V. I. Metody A. M. Lyapunova i ih primenenie. Moscow, Izdat. Leningrad. Univ., 1957. 241 pp. 12.35 rubles.

## MEMORANDUM TO MEMBERS

### A SPECIAL REQUEST CONCERNING THE ABSTRACTS OF CONTRIBUTED PAPERS PRESENTED IN PERSON

As you doubtless know, an attempt is being made to publish the abstracts of all the contributed papers presented in person at each meeting in the issue of the NOTICES which announces the meeting. At the same time there has been no timing change allowed in the publication schedule of the NOTICES. This publication schedule allows just ten working days after the abstracts deadline for preparation of camera copy in the Headquarters Offices.

It has been customary for many members to press the deadlines closely in the submission of abstracts. The January, 1958 meeting in Cincinnati was typical in this regard. The deadline was December 13, 1957. One hundred twenty-six abstracts of papers to be presented in person were received before the deadline. Of these, fifty-seven, or almost half, were received on the last two days, December 12 and 13. Thirty-seven, or twenty-nine per cent were received on the very last day, December 13. Seventy-three per cent of the papers were received during the week of the deadline.

It is the present feeling of the Editor of the NOTICES that if nearly one third of the in-person abstracts for a big annual meeting continue to come in on the day of the deadline, then the job of publishing all of them in the proper issue of the NOTICES might be impossible to accomplish within reasonable economic boundaries. The word "reasonable" can be construed here in the sense that the cost of publishing the abstracts in the NOTICES should not be more than the cost of publishing them by letterpress printing in the BULLETIN.

There are two obvious ways out of the dilemma. One is to request the Council to advance the deadlines. The other is to keep the present deadlines but to appeal to the members to send their abstracts in early.

The Editor is reluctant to ask for an advance in the deadlines because he knows from personal experience that the Society's present system of very late deadlines has advantages from the point of view of research workers and students. Therefore he now makes this appeal to all persons presenting papers in person at the meetings: PLEASE, IF YOU POSSIBLY CAN, SEND YOUR ABSTRACTS IN EARLY!

## ABSTRACTS OF CONTRIBUTED PAPERS

By direction of the Council, the abstracts of papers presented at the meetings of the Society are now being printed in the NOTICES instead of in the BULLETIN. An attempt is being made to publish the abstracts of all of the papers scheduled to be presented in person at each meeting in the issue of the NOTICES which carries the program of the meeting. There has been no advance in the deadlines for abstracts, but it is hoped that with a reasonable amount of cooperation from the members in sending abstracts in early, the project will be successful. Only abstracts which meet the specifications stated on the abstract blanks will be published.

It is necessary to use here a somewhat more mechanical editorial system of presentation than that which was formerly used in the BULLETIN. A brief description of the new system follows.

The numbering of the abstracts in the NOTICES is an arbitrary office numbering which is unrelated to the ordering of the papers in the programs. It does not distinguish between papers presented by title and papers presented in person. It does identify the meetings at which the papers will be, or were, presented. Thus the papers presented at the New York meeting in February of 1958, which is the five hundred and forty-third meeting of the Society, are numbered 543-1, 543-2, etc. There are cross references to these numbers in the programs. The abstracts are not arranged under subject classifications and there is no identification of the person presenting the paper in the case of joint authorship.

## THE OCTOBER MEETING IN WASHINGTON, D. C.

October 26, 1957.

538-1. M. A. Dengler: The decay of a vortex under the influence of viscosity.

Within two-dimensional fluid motion a vortex is defined as a thin cylindrical fluid filament which rotates about its own axis like a rigid body, with constant angular speed. The fluid velocity induced by the vortex varies as $1/r$ for $r > \epsilon$ (vortex radius) and as $r$ for $0 < r < \epsilon$. Placed into an ideal irrotational fluid, the vortex rotates indefinitely without losing its energy and, according to Kelvin, can not be destroyed. Placed into a real or viscous fluid, however, the vortex must decay due to viscous energy dissipation. The phenomenon of viscous vortex decay can be expressed by use of Bessel functions. Solutions approach zero everywhere, asymptotically, but depend on the initial conditions. The physically unrealistic assumption $v(r,0) = 1/r$ for $r > 0$ leads to the simplest type of solution. Exact solutions are, furthermore, derived for the following two sets of initial conditions: (a) $v(r,0) = (1 - e^{-r^2})/r$, $r \geqq 0$; (b) $v(r,0) = r$, $0 \leqq r \leqq 1$, $v(r,0) = 1/r$, $r \geqq 1$. (Received May 13, 1957.)

538-2. Joseph Bram: Chebychev approximation in locally compact spaces

The following generalization of Chebychev's theorem is proved. Let f be a real continuous function on X, vanishing at infinity. Let P be the linear span of n fixed real continuous functions $g_1,\ldots,g_n$ vanishing at $\infty$. Then n.a.s.c. that $p_0$ in P satisfy $\|f - p_0\| \leqq \|f-p\|$ for all p in P, where $\|g\| = \max\{|g(x)| : x \in X\}$, are that there exist an integer $r \leqq n$, and $r+1$ distinct points $x_0,\ldots,x_r$ in X such that the matrix $\{g_i(x_j)\}$, $i = 1,\ldots,n$; $j = 0,\ldots,r$ has rank r, and such that if the first r rows are independent and $D_j$ is the jth minor of the first r rows and $r+1$ columns, then $f(x_j) - p_0(x_j) = u \|f(x_j) - p_0(x_j)\| \cdot a_j$ for all $j = 0,\ldots,r$ such that $a_j \neq 0$, where $a_j = \operatorname{sgn}(-1)^j D_j$ and $u = \pm 1$. Uniqueness of $p_0$ does not hold, in general. (Received May 27, 1957.)

538-3. M. H. M. Esser: A characterization of $L^p$ spaces.

We give conditions under which a Banach space B is isomorphic to a space of measurable functions. We assume: (1) B contains a linearly dense Boolean ring (which becomes the set of characteristic functions); (2) the norm of

"orthogonal sums" is a nondecreasing function of the absolute value of the coefficients. Then (1) generates, as M. Stone has shown, a Boolean topological space X of maximal ideals (or, as we do it, after relaxing condition (1), of nested intervals), and (2) generates a subadditive measure on X, which we use to define convergence almost everywhere. This suffices for the representation of elements by functions on X. With some additional assumptions on the norm, we obtain an $L^p$ or an $L^\infty$ space. The complete treatment has been submitted to the Illinois J. of Mathematics. (Received June 11, 1957.)

538-4. J. W. Smith: Stability of quasi-linear difference equations.

The paper considers initial value problems for the difference equation (1) $u(x,t+\Delta t)=\sum_{r=-\rho}^{\rho} c^r(x,t,u)u(x+r\Delta x,t)+\Delta t d(x,t,u)$ applied over rectangular lattices covering $R(\tau_0)$: $-\infty < x < \infty$, $0 \le t \le \tau_0$. The notion of local stability is defined, and it is shown how local stability may be established by means of the fixed point theorem for n-space. The following results, which are extensions of Theorems 1.1 and 4.1 of Fritz John (Comm. Pure Appl. Math. vol. 5, 1952), have been obtained using Fritz John's techniques, supplemented by application of the fixed point theorem: (A) Let the coefficients of the differential equation (2) $u_t = a_0(x,t,u)u_{xx} + a_1(x,t,u)u_x + a_2(x,t,u)$ satisfy a Lipschitz condition in u, uniformly for $(x,t) \in R(\tau_0)$ and $|u|$ bounded and let u(x,t) be a solution of (2) such that u, $u_x$, $u_{xx}$ and $u_t$ are uniformly continuous and bounded on $R(\tau_0)$. The solutions $u(\Delta x)$ of (1), determined by initial values u(x,0), will converge locally to u as $\Delta x$ tends to 0, provided (1) is locally stable and compatible with (2). (B) Let the coefficients of (2) have mixed partial derivatives in x and u up to order 6 and first derivatives in t which are uniformly continuous and bounded, and let $a_0$ be positive and bounded away from zero, all for $(x,t) \in R(\tau_0)$ and $|u|$ bounded. Let u(x,0) belong to $C^6$ for $-\infty < x < \infty$. There exists a positive number $\tau \le \tau_0$ and a solution u(x,t) of (2), which belongs to $C^4$ on $R(\tau)$ and assumes the initial values u(x,0). (Received July 22, 1957.)

538-5. B. M. Dwork: Norm residue symbol in cyclotomic fields.

Let Q be the rational p-adic number field, Z be a primitive $p^r$-th root of 1, $K=K_r=Q(Z)$. The norm residue symbol is known by global methods: If n,m are integers, $nm \equiv 1 \bmod p^r$ then $(n,K/Q)Z=Z^m$. The author announces a purely local proof of this fact. The assertion is verified directly for $n=-1$, $p=2$. Let T be the p-adic completion of the compositum of all un-

ramified extensions of Q in the algebraic closure of Q, $A_i$ $(i = 0,1,\ldots)$ be a set of units in T which satisfy $A_0 = 1$, $\Delta A_{i+1} = A_{i+1} + A_i$, where $\Delta$ is the Frobenius substitution. Let s be a fixed generator of $G(K_r/K_2)$, $1 + t = (Z-1)^{s-1}$, $B = \sum_{i=0} A_i t^i$. It follows from a simple cohomological argument that it is enough to show that if $sZ = Z^n$ then $N_{KT/Q}\, B \equiv n \bmod p^r$. This is done by showing the norm to be $(\bmod p^r)$ $\prod_{\nu=1}^{\infty}[N_{K/Q}(1 + t^{p^\nu})]^{1/p^\nu}$. This reduction follows from the more general result (taking in this application $d = A_1$): If d is an integer in T then $(1+X)^d \prod_\nu^\infty{}_1 (1 + X^{p^\nu})^{d_\nu}$ is a formal power series in X whose coefficients are integers in T if $d_\nu = \Delta^{\nu-1}(\Delta - 1)d/p^\nu$. This more general result may be applied to norm residue computations for cyclic wildly ramified extensions of an arbitrary ground field. (Received July 23, 1957.)

538-6. James Radlow: Diffraction by a quarter-plane.

The three-dimensional problem of the diffraction of a plane wave by a quarter-plane leads to a two-variable singular integral equation, of the form $g(x_1,x_2) = \int_0^\infty \int_0^\infty f(x_1^0,x_2^0)k(|x_1 - x_1^0|, |x_2 - x_2^0|)dx_1^0\, dx_2^0$ where g and k are known, and f must be of exponential type; the equation is a generalization to two variables of the inhomogeneous Wiener-Hopf equation. We show how to apply the $L_2$ theory of double Laplace transforms, in particular results of Bochner (Amer. J. Math. vol. 59 (1937)), and Bergman-Martin (Duke Math. J. vol 6 (1940)) on the transform of a function in a quarter-plane, to construct a two-variable analogue of the classical function-theoretical argument of Wiener-Hopf. The two-variable factorization problem consists for us of expressing the transform $K(\zeta_1,\zeta_2)$ of the kernel $k(x_1,x_2)$ in the form $K(\zeta_1,\zeta_2) = M_1(\zeta_1,\zeta_2)/M_1^*(\zeta_1,\zeta_2)$ where $M_1$ is regular in a quarter-space $(\mathrm{Re}\,\zeta_k > a_k, -\infty < \mathrm{Im}\,\zeta_k < \infty;\ k = 1,2)$ while the factors of $M_1^*$ are regular in portions of the complementary three-quarter space. Expressions for $M_1$, $M_1^*$ are obtained, and the solution of our integral equation then follows readily. The final result is an explicit integral representation of the field of a plane wave in the presence of a quarter-plane. (Received July 24, 1957.)

538-7. J. S. Griffin, Jr. and J. E. McLaughlin: On the projective groups of lines.

Let D and E be division rings, let U and V be two dimensional vector spaces over D and E respectively, and let P and Q be the associated projective lines. It is shown that any 1-1 onto function f: $P \to Q$ which preserves projec-

tivities, i.e., such that $f\theta f^{-1} \in PGL_2(E)$ whenever $\theta \in PGL_2(D)$, is induced from a semi-linear isomorphism $\alpha: U \to V$, or else differs from such an $\alpha$ by a canonical map $\beta: V \to V^*$, where $V^*$ is the adjoint of V. Thus in particular D and E are either isomorphic or anti isomorphic. The proof is not difficult, and is based on the Cartan-Brauer-Hua theorem, and the theorem of Hua on semi-automorphisms of a division ring. (Received July 19, 1957.)

538-8. B. W. Jones and Sarvadaman Chowla: Note on perfect difference sets.

Let p denote a prime, and x a primitive element of the Galois field $GF(p^3)$ of $p^3$ elements. Let $x^{d_i} = x + i$ $(1 \leq i \leq p)$ where x is a primitive element of $GF(p^3)$. By Singer $d_0 = 0, d_1, d_2, \ldots, d_p$ form a perfect difference-set (mod $p^2 + p + 1$). By a theorem of Hall, p is a "multiplier" of this difference-set. We prove the following quantitative form of Hall's theorem: There exist constants a,b,c,d,e independent of i, all elements of GF(p), such that $d_i = pd_j + d_e$ where $j = (a+bi)/(c + di)$ (mod p) for every i for which $c + di \not\equiv 0 \pmod p$. (Received July 19, 1957.)

538-9. J. A. Hummel: A variational method for starlike functions.

The Schiffer variational method for the study of schlicht functions is not directly applicable to the class of starlike functions. If $w = f(z) = z + a_2 z^2 + \ldots$ maps $|z| < 1$ onto the starshaped domain D with boundary C, then the variation $w^* = w + \rho^2 w S(z)$ will map C onto the boundary $C^*$ of a starshaped domain if S(z) is real and bounded on $|z| = 1$, since then each point of C will be moved only radially. Schiffer has shown (Proceedings of the Symposia in Applied Mathematics, 1956, to appear) that under the variation $w^* = w + \rho^2 \delta w$, $\delta g(w,0) = \mathrm{Re}\{(\rho^2/2\pi i) \oint_C p'(\eta,w)p'(\eta,0)\delta\eta \, d\eta\} + o(\rho^2)$, where $g(w,\eta)$ is the Green's function of D and $p(w,\eta)$ is the analytic completion of $g(w,\eta)$. Using this and $S(z) = e^{i\theta}(1 - \bar{z}_0 z)/(z - z_0) + e^{i\theta}(z - z_0)/(1 - \bar{z}_0 z)$, a variational formula for f(z) in the class of starlike functions is easily computed. With the help of this formula, extremal problems in the class of starlike functions can be reduced to a differential equation of the form $[zf'(z)/f(z)]R(z) = Q(z)$ where R(z) and Q(z) are rational functions of z with Q(z) purely real and R(z) purely imaginary on $|z| = 1$. (Received July 29, 1957.)

538-10. J. F. Adams: On the nonexistence of elements of Hopf invariant one mod p.

The author has proved the following results in homotopy-theory: (1) It is not possible to form a complex $K = S^n \cup E^m$ such that $P_p^p: H^n(K;Z_p) \to H^m(K;Z_p)$ is nonzero. (2) The p-components of the stable r-stems for $r=2p(p-1)-2$, $r=2p(p-1)-1$ are $Z_p$, $Z_p$. The proofs depend on a Jacobi identity for stable higher Toda brackets. (Received August 2, 1957.)

538-11. P. J. Hilton and Walter Ledermann: Homology and ringoids. I.

MacLane (Bull. Amer. Math. Soc. vol. 62 (1956) p. 615) has indicated the desirability of presenting aspects of homological algebra exclusively in terms of the maps of the category. To this end we consider a category whose maps form a ringoid $\mathfrak{R}$ (Barratt, Quart. J. Math. (Oxford) vol. 5 (1954) p. 271); we impose on $\mathfrak{R}$ axioms derived from the Buchsbaum axioms for an exact category (Trans. Amer. Math. Soc. vol. 80 (1955) p. 1), but which are formulated entirely within the ringoid $\mathfrak{R}$ in the language of ring theory. We pick out the right-regular elements, $\epsilon$, of $\mathfrak{R}$ ('epimorphisms', defined by $\epsilon\alpha = 0 \Rightarrow \alpha = 0$; maps are written on the right) and the left-regular elements, $\mu$, of $\mathfrak{R}$ ('monomorphisms'). Let $R(\alpha)$ ($L(\alpha)$) be the right (left) ideal generated by $\alpha$. The axioms ensure that there exists a (1,1) correspondence $\{L(\mu)\} \leftrightarrow \{R(\epsilon)\}$ such that $L(\mu)$ is the left annihilator of $R(\epsilon)$ and $R(\epsilon)$ is the right annihilator of $L(\mu)$; we write $\mu A \epsilon$. Further each $\alpha \in R$ may be decomposed as $\epsilon\mu$. Then it follows that $\alpha$ determines $R(\epsilon)$ and $L(\mu)$, and that an element is right- and left-regular if and only if it is a unit. Moreover, for each $\alpha \in \mathfrak{R}$, the left (right) annihilator of $\alpha$ is a principal left (right) ideal. These axioms are sufficient for exact sequence and homology theory. Thus let $\alpha_1, \alpha_2 \in \mathfrak{R}$ and $\alpha_1\alpha_2 = 0$. Then if $\alpha_1 = \epsilon_1\mu_1$, $\alpha_2 = \epsilon_2\mu_2$ we have $\mu_1 \in L(\mu)$, where $\mu A \epsilon_2$. Then $\mu_1 = \beta\mu$, $\beta$ is left-regular and if $\beta A \eta$, then $\eta$ 'maps cycles to homology group'. (Received August 2, 1957.)

538-12. P. J. Hilton and Walter Ledermann: Homology and ringoids. II.

The ideas of note I are exemplified in the following ringoid $\mathfrak{M}$. Let $\mathfrak{A}$ be the category of finitely generated abelian groups, let $\mathfrak{P}$ be the category of 'presentations', that is, pairs of finitely generated free abelian groups F,R, with $R \subseteq F$, each endowed with a fixed basis, and let $\mathfrak{F}$ be the evident functor

$\mathcal{P} \to \mathcal{A}$. The basis of R is expressed in terms of the basis of F by means of a left-regular matrix, A; if matrices A and B are thus assigned to two objects p and q of $\mathcal{P}$, a map from p to q is described by an equation AT=SB. Let $\mathcal{M}_L^0$ be the category whose objects are left-regular matrices A and whose maps are quadruples (A,T,S,B) where AT= SB, composed by the rule (A,T,S,B)(B,U,V,C) = (A,TU,SV,C); and let $\psi_L^0$ be the evident functor $\mathcal{P} \to \mathcal{M}_L^0$. If $\alpha_i$ (i=0,1) is a map of $\mathcal{P}$ and $\alpha_1 \psi_L^0 = (A, T_1, S_1, B)$, then $\alpha_0 \Phi = \alpha_1 \Phi$ if and only if there exists X with $T_1 = T_0 + XB$, $S_1 = S_0 + AX$. Let $\mathcal{E}$ be the associated equivalence relation on $\mathcal{M}_L^0$ and let $\mathcal{M}$ be the quotient category $\mathcal{M}_L^0/\mathcal{E}$. Then $\psi_L^0$ induces $\psi: \mathcal{P} \to \mathcal{M}$; moreover the maps of $\mathcal{A}, \mathcal{P}, \mathcal{M}$ form ringoids $\mathcal{A}, \mathcal{B}, \mathcal{M}$ with respect to which $\Phi$ and $\psi$ are homomorphisms, and, if $\alpha$ is a map of $\mathcal{P}$, then $\alpha\Phi$ is an isomorphism if and only if $\alpha\psi$ is a unit of the ringoid $\mathcal{M}$. The ringoid $\mathcal{M}$ satisfies the axioms mentioned in I. (Received August 2, 1957.)

538-13. Marvin Shinbrot: Singularly perturbed integral equations.

An asymptotic expansion for the solution of the integral equation $\phi(x) = \int_{-1}^{1} k((x-t)/\epsilon)\, f(t)dt$, $|x| < 1$ as $\epsilon \to 0$ is derived. The application to the problem of diffraction of high frequency radiation through an infinite slit is discussed. (Received August 6, 1957.)

538-14. J. F. Vandrey: On the numerical evaluation of improper integrals.

Convergent improper integrals can be transformed into regular integrals by suitable substitutions. On this basis, a method is developed which gives approximate values for these integrals in the form of a weighted average of suitably selected ordinates. Formulas for the stations and their weights are given for a number of practically interesting cases. (Received July 24, 1957.)

538-15. W. S. Massey: On the cohomology ring of sphere bundles.

Let $p: E \to B$ be a fibre bundle with fibre a sphere of dimension k - 1. Denote the homomorphisms of the Gysin sequence by $\mu: H^q(B) \to H^{q+k}(B)$, $p^*: H^q(B) \to H^q(E)$, and $\psi: H^q(E) \to H^{q-k+1}(B)$ (see R. Thom, Ann. Ecole Norm. vol. 69 (1952) pp. 109-182). Assume the characteristic class vanishes; then $\mu = 0$, $p^*$ is an isomorphism into, and $\psi$ is onto. If a is any element of $H^{k-1}(E)$ such that $\psi(a) = 1$, then given any element $u \in H^q(E)$, there exist unique elements $x \in H^q(B)$ and $y \in H^{q-k+1}(B)$ such that $u = p^*(x) + a \cdot p^*(y)$. In particular, there exist elements $\alpha \in H^{2k-2}(B)$ and $\beta \in H^{k-1}(B)$ such that $a^2 = p^*(\alpha) + a \cdot p^*(\beta)$.

THEOREM 1. If k is even (resp. odd), then $\beta = W_{k-1}$ (resp. $\beta \equiv W_{k-1} \bmod 2$), where $W_{k-1}$ denotes a Stiefel-Whitney class. THEOREM 2. If k is odd, then $4\alpha + \beta^2 = P_{2k-2}$, a Pontrjagin class. These theorems enable one to deduce the structure of the cohomology ring $H^*(E)$ from that of $H^*(B)$ in many cases. (Received August 8, 1957.)

538-16. W. S. Massey: On the non-imbeddability of certain manifolds in Euclidean space.

For any integer n, let $P_n(C)$ and $P_n(Q)$ denote n-dimensional complex projective space and n-dimensional quaternionic projective space respectively. The following results are proved: (1) For $m > 1$, $P_{2m}(C)$ cannot be imbedded differentiably in Euclidean space of dimension $6m + 1$. (2) For $n > 2$, $P_n(Q)$ cannot be imbedded differentiably in Euclidean space of dimension $6n + 1$. (3) $P_7(C)$ cannot be imbedded differentiably in 21-dimensional Euclidean space. This last example is especially interesting because all the Stiefel-Whitney classes of $P_7(C)$ vanish. The proof of these results makes use of knowledge of certain Pontrjagin classes of the manifolds in question. (Received August 8, 1957.)

538-17. W. S. Massey: On the second obstruction in 2-sphere bundles.

Let $p : E \to B$ be a fibre bundle with fibre a 2-sphere $S^2$ and the set of all proper rotations of $S^2$ as group. Assume further that the base space B is a polyhedron, that the characteristic class vanishes (i.e., there exists a cross section defined over the 3-skeleton of B), and that the integral cohomology group $H^4(B)$ has no 2-torsion. THEOREM: There exists a cross section defined over the 4-skeleton of B if and only if there exists an integral cohomology class $\beta \in H^2(B)$ such that $\beta \equiv W_2 \bmod 2$ and $\beta^2 = P_4$, where $W_2$ and $P_4$ denote the Stiefel-Whitney class and Pontrjagin class respectively. The proof makes use of results of S. D. Liao (Ann. of Math. vol. 60 (1954) pp. 146-191). (Received August 8, 1957.)

538-18. A. J. Goldman and J. J. Stone: Continuous poker.

Two variants of the continuous poker games discussed by Von Neumann and Morgenstern in Theory of games and economic behavior (Princeton University Press, 1953, pp. 191-207, 211-219) are solved. Each allows an ante, bet and single raise (with seeing permitted) of fixed amounts. However one

variant calls for simultaneous bidding and is a "fair" game while the other calls for alternative bidding and is not "fair". The Von Neumann-Morgenstern games are limiting cases of these games. (Received August 9, 1957.)

538-19. Martin Schechter: On estimating partial differential operators. III

Let $L = \sum a_k \partial/\partial x_k$ be a linear first order partial differential operator with complex coefficients continuous in the closure $\bar{G}$ of a bounded domain G in n-space. L is elliptic if $\sum a_k \xi_k \neq 0$ for any set of real $\xi_k$ not all vanishing. Let $\gamma$ be a nonvanishing complex function defined on the boundary $\dot{G}$ of G and consider the set H of complex functions u continuously differentiable in $\bar{G}$ and such that $\operatorname{Im} \gamma u = 0$ on $\dot{G}$. Let $\|u\|$ denote the $L_2(G)$ norm of u and Du its generic first derivative. Then there is a constant K depending only on L, G, and $\gamma$ such that (1) $\|Du\|^2 \leq K(\|Lu\|^2 + \|u\|^2)$ for all $u \in H$ if and only if L is elliptic in $\bar{G}$. In particular (1) holds for the Cauchy-Riemann operator $L = \partial/\partial x + i\partial/\partial y$ and if we set $w_z = (w_x - iw_y)/2$, $w_{\bar{z}} = (w_x + iw_y)/2$, we have that $\|w_z\|^2 \leq K(\|w_{\bar{z}}\|^2 + \|w\|^2)$ for all complex functions w which are real on $\dot{G}$. (Received August 13, 1957.)

538-20. D. K. Harrison: Some notes on torsion-free Abelian groups.

Let S be all formal numbers $\prod p^{i_p}$, where all primes p appear, $i_p$ is an integer or $\infty$, and $i_p \neq 0$ for almost all p. S is ordered by "divides." A valuation of a vector space A over the rationals Q is a map $\phi: A \to S$ such that $\phi(a) = \prod p^{\infty}$ iff $a = 0$, $\phi(a+b) \geq \phi(a) \cap \phi(b)$, and $\phi(\alpha \cdot a) = \alpha \cdot \phi(a)$ for $\alpha \in Q, a, b \in A$. $\phi$ and $\theta$ are equivalent if $\exists\, g \in GL(A)$ such that $\phi \cdot g = \theta$. Equivalence classes of valuations are in a 1-1 correspondence with the reduced torsion free groups. The completions of A and $k = \{a \in A \mid \phi(a) \geq 1\}$ with respect to the valuations are isomorphic to Hom(Q,A/k) and Hom(Q/Z,A/k). Reduced groups k and h have isomorphic completions iff $f(k) = f(h)$ where $f(k) = \prod p^{f_p(k)}$ and $f_p(k) = \dim k/(p \cdot k)$. If rank $(k) < \infty$, this completion is a product of p-adic integers, $f_p(k)$ copies for each p. For k reduced that, k is complete, any torsion free extension of k is trivial, k is isomorphic to a direct summand of a direct product of p-adic integers, are equivalent. Let $e_p(k) = \dim \{a \in A \mid \phi(a) \geq p^{\infty}\}$, $e(k) = \prod p^{e_p(k)}$, and $n(k) = \prod p^{\operatorname{rank}(k)}$ Then $f(k) \cdot e(k) \leq n(k)$ If rank $(k) \leq \infty$ and $p \cdot k = k$ for almost all primes p, $f(k) \cdot e(k) = n(k)$ iff k is c.d. (completely decomposable), and pure subgroups of such groups are also c.d. Partial cohomology theory (see earlier report) is used. $2^{\aleph_0}$ groups of each

finite rank $\geq 2$ are not c.d. (Received August 19, 1957.)

538-21. Marvin Shinbrot: On the amalgamation of two methods for the solution of integral equations.

It is shown how the idea of Pearson (Quarterly of Applied Mathematics vol. 15 (1957) pp. 203-208) for the solution of the integral equation (i) $\phi(x) = \int_a^b k(x-t)f(t)dt$, $a<x<b$, when $k(x) = P(x)\log|x| + Q(x)$ where P and Q are polynomials, when put together with the method of Latta (J.Rat. Mech. and Anal. vol. 5 (1956) pp. 821 - 834) for the solution of (i) when k(x) satisfies a linear differential equation with linear coefficients, leads to the solution when (ii) $k(x) = P(x)m(x) + Q(x)$ where P and Q are polynomials and m satisfies Latta's differential equation condition. It is pointed out that since the range of $(x-t)$ in (i) is the bounded interval $[a-b, b-a]$, the method applies very well to approximate to solutions of (i) with kernels (ii) when P and Q are merely continuous. (Received August 19, 1957.)

538-22. L. P. Horwitz and H. H. Goldstine: A procedure for the diagonalization of normal matrices.

The authors extend the so-called Jacobi Procedure to the case of normal matrices. They describe a stable iterative procedure utilizing plane unitary transformations for such matrices which yield both the characteristic values and their associated vectors. Generally, the technique consists of minimizing at each stage the sum of the squares of the off-diagonal elements of the given matrix; however, there is one case in which this leads to no improvement; i.e. the lowest value for the change is non-negative. In this case, it is shown that a convergent procedure is still possible. (Received August 19, 1957.)

538-23. A. G. Laurent: On the "generalized correlation" between vectors. Preliminary Report.

Given $X = (X_1, X_2)'$, where $X_1, X_2$ are $p\times 1, q\times 1$ random vectors, $p \leq q$, with covariance matrix $\Sigma = (\Sigma_{ij})$, $i, j = 1, 2$, let $\Sigma_{kk} = \Sigma_k \Sigma_k'$, $\Sigma_k$ triangular and $\Xi_k = \Sigma_k X_k$, $k = 1, 2$. $\Xi = (\Xi_1, \Xi_2)'$ has covariance matrix $(P_{ij})$ where $P_{11}, P_{22}$ are identity matrices. The author (Ann. Math. Stat. Annual Meeting Abstracts, to appear) proposed to define the correlation between two vectors $X_1, X_2$ as $P_{21}$ and gave the distribution of the corresponding "correlation" $R_{21}$ based on sample values, when X is normally distributed and $P_{21} = 0$. Independently,

Healy (MATC. vol. 11, 1957, p. 84) proposed to diagonalize $R_{21}$ for computing canonical correlations. In the present paper, (a) partial and multiple correlations are generalized in a similar way, (b) some well known relationships are generalized to the vector case, (c) the distribution of $R_{21}$ is given when X is normally distributed and $P_{21} \neq 0$, (alternative hypothesis), (d) further developments are outlined, (distribution of canonical correlations, etc...). The generalized correlation has many applications in the field of multivariate analysis. (Received August 20, 1957.)

538-24. S. C. Kleene: Extension of an effectively generated class of functions by enumeration.

The analysis of a primitive recursive derivation of a function $\phi$ from an assumed function $\theta$, written in code form (say via prime-factor representation) as a number b, constitutes an index of $\phi$ from $\theta$; write $\phi$ then $\mathrm{pr}_b^{\theta}(a_1,\ldots,a_n)$. In this case, let $\mathrm{pr}^{\theta}(b,a) = \mathrm{pr}_b^{\theta}((a)_0,\ldots,(a)_{n-1})$; otherwise, $\mathrm{pr}^{\theta}(b,a) = 0$. (Primitive recursive) degree is defined by substituting primitive for general recursiveness in Kleene-Post, Ann. of Math. vol. 59 (1954) p. 381. Formulas (1) - (12) there hold for these degrees, when $\underline{0}$ is the degree of a primitive recursive function, $\cup$ is as there, and $\underline{a}'$ is the degree of $\mathrm{pr}^{\theta}(b,a)$ for $\theta$ of degree $\underline{a}$. The recursion theorem holds for primitive recursive functions and indices. The system O of notations for the ordinals $< \omega_1$ (e.g. Kleene, Trans. Amer. Math. Soc. vol. 79 (1955) pp. 324-325) is modified to use only primitive recursive fundamental sequences of notations, described by indices. A hierarchy of general recursive functions $h_y(b,a)$, for $y \in$ this O, is defined in the same manner (with $h_y(b,a) = \mathrm{pr}^{h_z}(b,a)$ for $y = 2^z \neq 1$) as the hierarchy of predicates $H_y(a)$ in Kleene loc. cit.; for $y <_O u$, $h_y$ is of lower degree than $h_u$. (Received August 23, 1957.)

538-25. P. D. Lax: The eigenvalues as monotonic matrix functions.

Consider a linear space of real matrices every element of which has only real eigenvalues. It follows from a previous result of the author (Bull. Amer. Math. Soc. Abstract 63-2-235) that the set of elements all of whose eigenvalues are positive form a convex cone. Here we show that in the sense of partial ordering with respect to these positive matrices, all eigenvalues are monotonic increasing functions. This generalizes a classical theorem on symmetric matrices. This result is used to show the stability of matrix difference oper-

ators with positive coefficients, generalizing a criterion due to Friedrichs. The proof relies on the theory of hyperbolic differential equations. (Received August 23, 1957.)

538-26. M. P. Schützenberger: On a representation of commuting monoids.

Let $(\Sigma, S)$ be a representation of the monoids S by mappings of $\Sigma$ into itself; $\{P_i\}$ $(i \in I)$ a family of submonoids of S that commute two by two elementwise; $L_i$, the preorder on $\Sigma$ defined by $\sigma L_i \sigma'$ when $\sigma \in \sigma' P_i$. $\overline{L}_i = L_i \cap L_i^{-1}$; if $\phi \neq I' \subset I$, $\overline{L}_{I'} = \bigcap \{\overline{L}_i : i \in I'\}$. $1^\circ$. For all $i,i' \in I$: $\overline{L}_i \circ \overline{L}_{i'} = \overline{L}_{i'} \circ \overline{L}_i$; $L_i \circ L_{i'} = L_{i'} \circ L_i$; $L_i \circ L_{i'}^{-1} \subset L_{i'}^{-1} \circ L_i$. Let $\Xi \subset \Sigma$ be a not empty $\overline{L}_{I'}$-class; $P_{i'} = \{x \in P_i : \Xi x \cap \Xi \neq \phi\}$; $\phi$, the homomorphism of $P_{i'}$ induced by the restriction of $\Sigma$ to $\Xi$. $2^\circ$. If $i \in I' \neq \{i\}$, $\phi P_i$ is a group $G_i$ and if $i,i' \in I'$ $(i \neq i')$ $G_i$ and $G_{i'}$ are anti-isomorphic and, consequently, abelian when I' has more than two elements. $1^\circ$ is a generalization of results of J. A. Green (Ann. Math. vol. 54 (1951) pp. 163-172), $2^\circ$ of Bull. Amer. Math. Soc. Abstract 63-4-510. (Received August 21, 1957.)

538-27. R. C. Buck: The Riesz property of a partially ordered linear space.

Let V be a real linear space with positive cone P. Assume that $P \cap -P = \{0\}$ and that P and -P generate V. Let $V^*$ be the dual space of linear functionals, and $P^*$ the dual cone of positive linear functionals. It is easily seen that $P^* \cap -P^* = \{0\}$. V has the Riesz property (Namioka, Memoirs Amer. Math. Soc. no. 24) if whenever x, y, and z are in P and $x + y \geqq z$, then $z = x_0 + y_0$ with $0 \leqq x_0 \leqq x$ and $0 \leqq y_0 \leqq y$. If V has the Riesz property, then $P^*$ and $-P^*$ together generate the subspace of bounded functionals. It is known that a vector lattice has the Riesz property. THEOREM. V has the Riesz property if and only if V is a weak lattice. (Roughly speaking, this means that any two lower bounds for a pair of elements can be replaced by a single lower bound which is larger than either.) (Received August 26, 1957.)

538-28. R. C. Buck: The algebra of bounded analytic functions.

Let D be the open unit disc, and B be the algebra of all bounded analytic functions on D. A study is made of the strictly continuous linear functionals on B, and applied to the classification of strictly closed ideals in B. The dual space B' can be represented as a class of functions h(w) analytic in D. For any

$f \in B$, the Hadamard product f o h is a function which is continuous in the closed disc $\overline{D}$, and the corresponding functional value is (f o h) (1). B' contains the space $H_\infty$. Polynomials and canonical Blaschke products generate closed principal ideals, but not all principal ideals are closed. Conjecture: if $g \in B$ and g has no zeros in D, then (g) is dense. This holds (e.g.) if Re(1/g) is bounded from below in D, or if g is analytic in $\overline{D}$, or if $B/(g)^-$ obeys the ascending chain condition for closed ideals. However, there is a function $g \in B$ without zeros such that 1 is not a limit point of Sg, for any bounded set S. (The strict topology was introduced by the author in a study of the bounded continuous functions on locally compact spaces; see Proc. Amer. Math. Soc. vol. 3 (1952) pp. 681-687.) (Received August 26, 1957.)

538-29. M. D. Burrow: Invariants of a free Lie ring.

A free Lie ring L with n free generators has no invariant of degree 2n under the unimodular group if n = 2 or 3 (W. Magnus, J.Reine Angew. Math. vol. 182 (1940) pp. 142-149; F. Wever, Math. Ann. vol. 120 (1949) pp. 563-580). This paper settles the question for arbitrary n by constructing for any $n > 3$ an invariant of degree 2n. Suppose $\xi_i$, $i = 1,\ldots, n$ generate L and let $f = \det (x^j_{\pi(i)}) \det (x^j_{\sigma(i)})$ where the $x^j_k$ are algebraic indeterminates and $\pi, \sigma$ are functions whose disjoint ranges span $1,\ldots,2n$. Now if $z_s$ is the permutation $(12\ldots s)$ and $\Omega_{2n} = (1 - z_{2n})\ldots(1 - z_2)$ then $t = \Omega_{2n} \cdot f$ if nonvanishing corresponds to an invariant $\theta \in L$, constructible from t, under the mapping $\xi_i \rightarrow x^i_1$, $(\xi_i, \xi_j) \rightarrow x^i_1 x^j_2 - x^j_1 x^i_2$ etc.; here the $x^k$ in each product are sub-indexed by position. By studying the effects of those terms of $\Omega_{2n}$ which belong to the product of the row and column permutation groups of a certain $n \times 2$ array $\pi(i)\,\sigma(i)$ it is shown that $t \neq 0$. Similarly an invariant of degree $n^2$ can be constructed for any $n > 3$. (Received August 26, 1957.)

538-30. R. S. Palais: Extending germs of diffeomorphisms.

Let M be a $C^\infty$ manifold and $x \in M$. Let D(x,M) be all mappings defined in an open neighborhood of x and mapping this neighborhood diffeomorphically onto some open set of M. Call two elements of D(x,M) equivalent if their restrictions to some neighborhood of x are equal. Let G(x,M) be the set of equivalence classes of D(x,M) under this relation. The elements of $\bigcup_{x \in M} G(M,x)$ are called germs of diffeomorphisms of M. A germ of a diffeomorphism of M is called extendible if it contains a diffeomorphism of all of M with itself. Say that

M is reversible if M is orientable and there exists an orientation reversing map of M. The following theorem is proved: If M is nonorientable or reversible then any germ of a diffeomorphism of M is extendible. If M is orientable and not reversible then a germ of a diffeomorphism of M is extendible if and only if it is orientation preserving. This follows from the following analytical lemma: If $f_i$ are $nC^\infty$ real valued functions defined in a neighborhood of the origin of $R^n$ such that $f_i(0) = 0$ and $(\partial f_i/\partial x_j)_0 = \delta_{ij}$ then for all sufficiently small positive t there exists a diffeomorphism $\phi$ of $R^n$ such that $\phi(x)_i = f_i(x)$ for $\|x\| < t$ and $\phi(x) = x$ for $\|x\| > 2t$. (Received August 26, 1957.)

538-31. R. S. Palais: Natural operations on differential forms.

Let M be an n-dimensional $C^\infty$ manifold, $\mathfrak{F}^p(M)$ the real vector space of $C^\infty$ p-forms on M, $\mathfrak{F}(M) = \oplus_{p=0}\mathfrak{F}^p(M)$, $E_p$ the projection of $\mathfrak{F}(M)$ on $\mathfrak{F}^p(M)$, and d the exterior derivative. Let G be the group of self diffeomorphisms of M. Each $g \in G$ induces in a well known way an automorphism $R_g$ of $\mathfrak{F}(M)$, and $R: g \to R_g$ is a representation of G in $\mathfrak{F}(M)$. Let N be the commuting ring of this representation, that is to say all linear mappings of forms into forms which are 'natural' in the sense that they commute with all $R_g$. Clearly the $E_p$ belong to N and $N_{pq} = E_p N E_q$ gives a decomposition of N into direct summands. It is shown that for $p > 0$, $N_{pp}$ contains only constant multiples of $E_p$, $N_{p,p+1}$ contains only constant multiples of $d^p = dE_p$, and the other $E_{pq}$ contain only zero. The nature of the $E_{0q}$ is elucidated but not completely determined. The proofs are based on the observation that for $p > 0$, $N_{pq}$ contains only localizeable operations (i.e. operations $\Theta$ such that $\phi = \psi$ in an open set $\mathcal{O}$ implies $\Theta(\phi) = \Theta(\psi)$ in $\mathcal{O}$). Together with the preceding abstract this reduces the problem to a local one which is solved by using local canonical forms for differential forms. (Received August 26, 1957.)

538-32. Paul McDougle: A theorem on quasi-compact mappings.

Let $f(X) = Y$ where f is continuous, and X and Y are topological spaces. Then f is semi-closed provided that f(C) is closed for each compact subset C in X; f is a $P_2$ mapping provided that for each y in Y there is a compact subset C of $f^{-1}(y)$ such that y is interior to f(U) for every neighborhood U of C. The following theorem is proved: If f is a quasi-compact mapping of a locally compact, separable metric space X onto a topological space Y, then Y is locally compact, separable metric if and only if f is a semi-closed, $P_2$ mapping.

(Received August 26, 1957.)

538-33. D. E. Spencer: On generalized Newtonian mechanics.

In the new electrodynamics (Moon and Spencer, J. Franklin Inst. vol. 257 (1954) p. 369), an equation was developed for the force between moving charges, as a function of their separation, relative velocity, and acceleration. This development provides an alternative to Maxwell's equations and does not use Einstein's relativity. A similar extension is proposed for Newton's laws. The proposed equation for the force between any two masses contains two terms. The first is a generalization of Newton's gravitational force, modified by a function of distance which permits the interpretation of the galactic red-shifts as actual motions. The second term is directly proportional to acceleration. The term contains a coefficient which is inversely proportional to distance, and also a function of the velocity relative to the substratum. When integrated over the cosmos, this term gives the inertial force. Achievements which result are: a mathematical formulation of Mach's principle, the limitation of the velocity of matter to c relative to the substratum, an alternative to the cosmological constant of Einstein, and a correct evaluation of the advancé of the perihelion of Mercury. The equation is also of significance for the formulation of Newtonian cosmologies. Galilean relativity is employed throughout. (Received August 28, 1957.)

538-34. Henry Hiz: Extendable sentential calculus.

The system S of the sentential calculus based on the rules: I. $\alpha \supset \beta$, $\beta \supset \gamma \Rightarrow \alpha \supset \gamma$ ; II. $\alpha \supset (\beta \supset \gamma)$, $\alpha \supset \beta \Rightarrow \alpha \supset \gamma$ ; III. $\sim \alpha \supset \beta$, $\sim \alpha \supset \sim \beta \Rightarrow \alpha$, and on the axioms: IV. $\sim(\alpha \supset \beta) \supset \alpha$ ; V. $\sim(\alpha \supset \beta) \supset \sim\beta$ is complete (every logically true formula is a theorem). S is not Post-complete. S is $\omega$-extendable: there is an infinite sequence $\{\alpha_i\}$ of nonequivalent formulas such that addition of any $\alpha_n$ to S leaves all $\alpha_m (m < n)$ unprovable. Meta-S is incomplete: modus ponens is not a derivable rule of inference. (Received September 3, 1957.)

538-35. H. F. Weinberger: Lower bounds for higher eigenvalues by finite difference methods.

It was independently shown by J. Hersch (C. R. Acad. Sci. Paris vol. 240 (1955) pp. 1602-1604) and the author (Bull. Amer. Math. Soc. vol. 61 (1955) p. 30

and Comm. Pure Appl. Math. vol. 9 (1956) pp. 613-623) that a lower bound for the lowest eigenvalue $\lambda_1$ of the Laplace operator on a bounded two-dimensional domain R with zero boundary condition can be given in terms of the lowest eigenvalue $\lambda_1^{(h)}$ of the analogous finite difference problem on a mesh of size h on a slightly larger domain $R_h$. This result is now extended to all eigenvalues $\lambda_k$ of the general second order elliptic self-adjoint eigenvalue problem with zero boundary condition on a bounded N-dimensional domain R. The lower bound is of the form $\lambda_k^{(h)} - f(h, \lambda_k^{(h)})$, where $\lambda_k^{(h)}$ is the kth eigenvalue of an analogous finite difference problem on a mesh of size h on a slightly larger (by a distance of order h) domain $R_h$. The function f can be computed explicitly, and depends only on the coefficients of the equation and the domain R, but not on k. In general it is of order $h^{1/2}$ as $h \to 0$. If the differential equation contains no mixed derivatives, it is of order $h^2$. (This work was sponsored by the U. S. Army under Contract No. DA-11-022-ORD-2059.)(Received September 3, 1957.)

538-36. H. J. Ryser: Inequalities of compound and induced matrices with applications to combinatorial analysis.

Let H be a nonnegative hermitian matrix of order $v > 1$, with characteristic roots $\lambda_1 \geqq \ldots \geqq \lambda_v$. $C_r(H)$ denotes the r-th compound or adjugate matrix of H, $P_r(H)$ the r-th induced or power matrix of H, tr(H) the trace of H. Let k and $\lambda$ satisfy (1) $\mathrm{tr}(H) = kv$ and (2) $\lambda_v \leqq k + (v-1)\lambda \leqq \lambda_1$. Let B be the matrix of order v with k in the main diagonal and $\lambda$ in all other positions. Then (3) $\mathrm{tr}(C_r(H)) \leqq \mathrm{tr}(C_r(B))$. If equality holds in (3) for an $r > 1$ and $k + (v-1)\lambda \neq 0$, then there exists a unitary U such that $H = U^{-1}BU$. Let $\mu$ be the sum of all of the elements of H and set (4) $\mu = (k + (v-1)\lambda)v$. The $\lambda$ defined by (4) satisfies (2). Moreover, for this choice of $\lambda$, if equality holds in (3) for an $r > 1$ and $k + (v-1)\lambda \neq 0$, then $H = B$. Analogous results hold for power matrices, where for this case $\mathrm{tr}(P_r(H)) \geqq \mathrm{tr}(P_r(B))$. Applications to incidence matrices of v, k, $\lambda$ configurations are studied in detail, and the author's Theorem 3 on maximal determinants is obtained as a special case of a more general result (Canadian J. Math. vol. 8 (1956) pp. 245-249). (Received September 4, 1957.)

538-37. L. A. Rubel: Extreme limits and Tauberian theorems.

For indeterminate forms with suitably restricted numerator and denominator, the author establishes a version of L'Hopital's rule, together with an appropriate converse, in which the order of derivation may be any real number

$p, -1 < p \leqq \infty$. The theory is developed as a parallel of Polya's theory of maximal and minimal densities. Littlewood's celebrated Tauberian theorem is a corollary of the principal theorem. For a fixed bounded measurable function $f$ and $\xi \in [0,1)$, let $L(\xi) = \lim\sup_{x\to\infty}(x - \xi x)^{-1}\int_{\xi x}^{x} f(t)dt$, $\ell(\xi) = \lim\inf$ (same). For $-1 < p < \infty$, define $F^{(p)}(x) = (-)^p \int_0^\infty t^p e^{-xt} f(t)dt$, $I^{(p)}(x) = (-)^p \int_0^\infty t^p e^{-xt}dt = (-)^p \Gamma(p+1)x^{-(p+1)}$. Define $\Lambda(p) = \lim\sup_{x\to 0+} F^{(p)}(x)/I^{(p)}(x)$, $\lambda(p) = \lim\inf$ (same). THEOREM 1: $L(1)$, $\ell(1)$, $\Lambda(\infty)$, and $\lambda(\infty)$ exist and $\ell(1) \leqq \ell(\xi) \leqq L(\xi) \leqq L(1)$, $\lambda(\infty) \leqq \lambda(p) \leqq \Lambda(p) \leqq \Lambda(\infty)$. THEOREM 2: $f$ is Cesaro (Abel) summable if and only if $L(\xi) = \ell(\xi)$ ($\Lambda(p) = \lambda(p)$) for all $\xi$ (all $p$). THEOREM 3: $L(1) = \Lambda(\infty)$, $\ell(1) = \lambda(\infty)$. THEOREM 4: (Littlewood) A bounded $f$ is Abel summable if and only if it is Cesaro summable. (Received September 5, 1957.)

538-38. F. A. Homann: Projective geometrical aspects of Laguerre-Forsyth canonical form.

Let C be a curve immersed in n-1 dimensional projective space $S_{n-1}$ and defined by n linearly independent solutions $x_1(t),\ldots,x_n(t)$ of an nth order homogeneous linear differential equation in Laguerre-Forsyth canonical form. Both C and the osculating subspaces $S_k$ ($k=0,1,\ldots,n-1$) at each point $x(t)=(x_1(t),\ldots, x_n(t))$ of C are invariant under the four-parameter continuous group $\{T\}$ of transformations leaving the equation in canonical form. But the transform $T(x/k)$ of the kth derivative point $x/k$ ($k=1,2,\ldots,n-1$) traces a locus C' in the $S_k$ at $x(t)$ as T is varied continuously. Differentiation of the transformation formulas yields homogeneous coordinates of $T(x/k)$ in $S_k$ as continuous functions of a variation parameter $\omega$. Elimination of $\omega$ gives a set of equations defining surfaces whose intersection is C'. Also, C' coincides in the limit as $\epsilon \to 0$ with the locus of the point of intersection y of the $S_k$ at $x(t)$ and the $S_{n-1-k}$ of a nearby point $x(t+\epsilon)$ of C. This is proved by determining the homogeneous local coordinates in $S_k$ of y as continuous functions of $\epsilon$ and eliminating $\epsilon$ to get the previous set of surfaces. (Received September 5, 1957.)

538-39. R. Z. Norman and M. O. Rabin: An algorithm for a minimum cover of a graph.

A set C of edges of a graph G is a cover of G if every vertex of G is incident to an edge of C. An alternating path of (G,C) is a sequence of distinct edges which are successively adjacent and are alternately in and not in C. A

reducing path is an alternating path whose terminal edges are in C and whose terminal vertices are incident to edges of C other than the terminal edges. THEOREM: C is a minimum cover of G if and only if (G,C) has no reducing path. This gives rise to an algorithm for transforming a given cover to one with a minimum number of edges. This minimization problem is the one-dimensional instance of the minimum cover problem of J. P. Roth (Algebraic Topological Methods for the Synthesis of Switching Systems I, to appear in Trans. Amer. Math. Soc.). A set D of edges of G is a disjoint set if no two edges of D have a common vertex. An augmenting path of (G,D) is an alternating path with distinct vertices whose terminal vertices are not incident to any edges of D. Claude Berge has shown (Proc. Nat. Acad. Sci., Sept. 1957) that D is a maximum disjoint set if and only if (G,D) has no augmenting path. Both theorems, originally proved by induction, are now proved by minimizing a distance function defined on the space of all sets of edges of G. The relation between these problems is discussed. (Received September 5, 1957.)

538-40. J. A. Kalman: Complemented lattices in universal algebra.

THEOREM. Let A be an algebra (in the sense of Birkhoff, Lattice theory, rev. ed., p. vii), and let the lattice of all subalgebras of A which contain the fixed subalgebra B be complemented; then B coincides with the intersection of all maximal proper subalgebras of A which contain B. Corollary. The $\Phi$-subgroup of an arbitrary group with complemented subgroup lattice consists of the identity only (cf. Suzuki, Structure of a group and the structure of its lattice of subgroups, Berlin, 1956, p. 26). THEOREM. An algebra with complemented structure lattice is isomorphic with a subdirect union of simple algebras (cf. Blair, Trans. Amer. Math. Soc. vol. 75 (1953) p. 138; Birkhoff, op. cit., p. 92). An example shows that the complete direct union of two simple algebras need not have complemented structure lattice. The above theorems follow from the purely lattice-theoretic Theorem. In a complemented inductive closure system the set of all neutral elements coincides with the intersection of all ultra-ideals (for terminology cf. Schmidt, Ber. Math. Tagung Berlin, 1953, pp. 21-48). This theorem may be deduced from the generalized Frattini-Neumann theorem of Schmidt (op. cit., p. 36). (Research supported by the University of New Zealand Research Fund.) (Received September 6, 1957.)

538-41. H. I. Levine: An order function for analytic mappings of complex manifolds into projective space. Preliminary report.

Let f be an analytic mapping of an r complex dimensional complex manifold, M, into $P_N$, complex projective N space, and let D be an open subset of M whose closure is compact and whose boundary $D^*$ is a 2r-1 dimensional submanifold of M. Let A be a linear subspace of $P_N$ of dimension N-r such that the points of D whose images under f lie on A are isolated. The number of such points, counting multiplicities, is denoted by $_A(D)$. Let $\Omega$ be the fundamental exterior differential 2-form associated with a Kähler metric on $P_N$. Define the order function, t(D) of f, by $t(D) = k \int_D f^*(\Omega^r)$, where $\Omega^r = \Omega \wedge \Omega \wedge \ldots \wedge \Omega$ (r times), and k is a constant such that $k \int \Omega^r = 1$ when the integral is extended over a linear subspace of $P_N$ of dimension r. It is found that $n_A(D) + m_A(D^*) = t(D)$ where $m_A(D^*)$ is an integral over $D^*$ which depends on A, f, $D^*$, and the metric chosen in $P_N$. This is a generalization of the first theorem of equidistribution of Ahlfors and the Weyls (H. Weyl, Meromorphic functions and analytic curves, Princeton, 1943). Further it is found that the average over all A of $m_A(D^*)$ is zero. The formula when applied to compact M gives a proof of the fact that $n_A(M)$ is independent of A and is equal to t(M). (Received September 6, 1957.)

538-42. L. B. Robinson: Short proof of a theorem of Riquier.

The author has been working tensors and affinors basing his work on that of Riquier. Now Riquier gives us the following theorem: Réciproquement, toute solution ordinaire du système (I) peut s'obtenir en égalant à zero trois solutions convenablement choisies du système (4) et résolvant le système ainsi obtenu par rapport à u, v, w conformément au principe général des fonctions implicites (Les Systèmes D'Équations aux Dérivées Partielles, p. 506). But it took him five and one half pages to demonstrate this theorem. The author has found a much shorter proof. He then proceeds to show that his work has a bearing on metamathematics. (Received September 6, 1957.)

538-43. Withdrawn.

538-44. A. N. McCauley: On the existence of real eigenvalues for a non-self-adjoint boundary-value problem.

Consider the vector-matrix differential system $\dot{x} = (\lambda A + B)x$ where

$A = (a_{ij})$ and $B = (b_{ij})$ are $2 \times 2$ real matrices which are continuous in t on [0,1], $\dot{x} = dx/dt$, $x = \mathrm{col}(x_1, x_2)$, and $\lambda$ is a parameter. Call $\Lambda(t)$ the minimum eigenvalue of $A_\theta = (\theta_{ij})$, $\theta_{11} = -a_{21}$, $\theta_{22} = a_{12}$, $\theta_{12} = \theta_{21} = (a_{11} - a_{22})/2$ and assume that $\int_0^1 \Lambda(t)dt > 0$. Then, there exist an infinite number of real eigenvalues, $u_n$ $(n = 0, \pm 1, \pm 2, \ldots)$ in the case of each of the following boundary conditions: I (Sturmian) $x_1(0)\cos\alpha - x_2(0)\sin\alpha = 0$, $x_1(1)\cos\beta - x_2(1)\sin\beta = 0$ where $\alpha$ and $\beta$ are preassigned, $0 \leq \alpha < \beta \leq \pi$; II (Periodic) $x(0) = x(1)$ if $\int_0^1 \mathrm{Tr}\, A dt = \int_0^1 \mathrm{Tr}\, B dt = 0$; and III (Mixed) $x(0) = Px(1)$ if $\int_0^1 \mathrm{Tr}\, A dt = \int_0^1 \mathrm{Tr}\, B dt = 0$ and the constant $2 \times 2$ matrix P has determinant equal to 1. The proof, in outline, for the system with Sturmian conditions is as follows. A solution, $x = \phi(t, \lambda)$, of $\dot{x} = (\lambda A + B)x$ exists satisfying $\phi_1(0, \lambda) = \sin\alpha$ and $\phi_2(0, \lambda) = \cos\alpha$ by the usual existence theorems. From a consideration of an expression involving $A_\theta$, the continuous function $\omega(1, \lambda) = \phi_1(1, \lambda)/\phi_2(1, \lambda)$ is shown to vary from negative to positive infinity as $\lambda$ runs from negative to positive infinity, respectively. Therefore, $\omega(1, \lambda)$ takes on the values $\beta + n\pi$ $(n = 0, \pm 1, \pm 2, \ldots)$ and the second of the two Sturmian boundary conditions is clearly satisfied at these values. The proofs for the periodic and mixed boundary conditions require the existence of the eigenvalues afforded by the Sturmian case. (Received September 9, 1957.)

538-45. C. T. Taam: A note on nonlinear oscillations.

Consider $x'' + px + qx^3 = f$, where p(t), q(t) and f(t) are real-valued, bounded periodic Lebesgue-measurable functions with a common least period L. It is further assumed that p(t) and q(t) are even functions with positive lower bounds and f(t) an odd function with a non-negative lower bound on (0,L/2). In a previous paper (Bull. Amer. Math. Soc. Abstract 63-2-263) several criteria for the existence of periodic solutions, harmonic and subharmonic of order n, as well as their oscillatory properties were reported. This present note contains improvements of some of these results. (Received September 9, 1957.)

538-46. Fred Supnick and H. J. Cohen: On the powers of a real number reduced modulo one.

Let (x) denote the fractional part of the real number x, and consider the sequence $(\alpha), (\alpha^2), (\alpha^3), \ldots$, where $\alpha$ is a real number $> 1$. The authors ask: Are the elements of this sequence distinct for a given $\alpha$? The relation R on an arbitrary set $\mathcal{E}$ of positive integers is defined by: iRj if and only if $\alpha^i - \alpha^j$ is integral. Then, R is an equivalence relation and induces a decomposition of $\mathcal{E}$

into equivalence classes: $\mathcal{E} = C_1 + C_2 + C_3 + \dots$. $\mathcal{E}$ is infinitely decomposed by if there are infinitely many $C_i$, and maximally decomposed by $\alpha$ if each $C_i$ contains just one element (e.g. if $\alpha$ is transcendental, or nonintegral rational). It is shown: If $\alpha^t$ is rational, but nonintegral, for some positive integer t, then the set $\mathcal{A}$ of all positive integers is maximally decomposed. If h is the smallest positive integer such that $\alpha^h$ is integral, then the set $\mathcal{A} - \{2h, 3h, 4h, \dots\}$ is maximally decomposed. If $\alpha^q$ is irrational for all positive integral q, then any arithmetic progression $\mathcal{E}$ is infinitely decomposed. Other cases are given where certain algebraic conditions on $\alpha$ imply that $\mathcal{A}$ is maximally decomposed. (Received September 9, 1957.)

538-47. Allen Devinatz and I. I. Hirschman, Jr: The spectra of multiplier transforms on $\ell^p$.

Let $\ell^p$ be the usual sequence space, let $\ell_0$ be the space of those sequences F(n) which are zero except for a finite set of indices n, and let $\ell$ be the space of all sequences F(n). A linear transformation T of $\ell_0$ to $\ell$ is said to be a multiplier transformation if $(TF)^*G = F^*(TG)$ for every F, G in $\ell_0$. Let I(T) be the set of indices p for which T can be extended to be a bounded operator on $\ell^p$ to itself. I(T) is an interval and it is well known that there exists a bounded measurable function $T^\wedge(\theta)$ on [0,1] such that $(TF)(n) = \int_0^1 F^\wedge(\theta) T^\wedge(\theta) \exp(-2\pi i n\theta) d\theta$ if $F \in \ell^2 \cap \ell^p$. Here $F^\wedge(\theta) = \sum_{-\infty}^{\infty} F(n)\exp(2\pi i n\theta)$. It is shown that if $T^\wedge(\theta+)$ and $T^\wedge(\theta-)$ exist for all $\theta$ and if p is an interior point of I(T) then the spectrum of T coincides with the essential range of $T^\wedge(\theta)$. Let $p > 2$. We set $\mathcal{E}_\lambda^\epsilon = \{\theta \mid |T^\wedge(\theta) - \lambda| < \epsilon\}$ and $\mathcal{E}_\lambda = \bigcap_{\epsilon>0} \bar{\mathcal{E}}_\lambda^\epsilon$. It is shown that under the above assumptions on T and p if $TF = 0$ ($F \in \ell^p$) then $F^\wedge(\theta) = 0$ on the complement of $\mathcal{E}_\lambda$. Conversely if $T^\wedge(\theta)$ is of bounded $\beta$ variation, $F \in \ell^p$, and $F^\wedge(\theta) = 0$ on the complement of $\mathcal{E}_\lambda$, then $(T - \lambda I)^m F = 0$ if m is an integer satisfying $m \geq \beta(p-2)/2p$. $T^\wedge(\theta)$ is said to be of bounded $\beta$ variation if $\sup \sum_1^n |T^\wedge(\theta_{n+1}) - T^\wedge(\theta_n)|^\beta < \infty$ for all $0 \leq \theta_1 < \theta_2 < \dots < \theta_{n+1} < 1$. (Received September 9, 1957.)

538-48. Allen Devinatz and I. I. Hirschman, Jr: Multiplier transforms on $\ell^{2,\alpha}$.

Let $\ell^{2,\alpha}$ $(-1/2 < \alpha < 1/2)$ be the space of those functions F(n) such that $\|F\|_{2,\alpha} = \{\sum_{-\infty}^{\infty} |F(n)|^2 (|n|+1)^{2\alpha}\}^{1/2}$ is finite. Let $\ell_0$ be the space of those sequences F(n) which are zero except for a finite set of indices n, and $\ell$ the space

of all sequences F(n). A linear transformation T of $\ell_0$ to $\ell$ is said to be a multiplier transform if TF*G = F*TG for every F,G in $\ell_0$. Suppose that T can be extended so as to be a bounded transformation of $\ell^{2,\alpha}$ into itself for some fixed $\alpha$. It is easily seen that there exists a bounded measurable function $T\hat{}(\theta)$ on [0,1] such that $(TF)(n) = \int_0^1 F\hat{}(\theta)T\hat{}(\theta)\exp(-2\pi in\theta)\,d\theta$, $F \in \ell^{2,\alpha} \cap \ell^{2,\rho}$. Here $F\hat{}(\theta) = \sum_{-\infty}^{\infty} F(n)\exp(2\pi in\theta)$. Let R be the essential range of $T\hat{}(\theta)$. It is shown that if $w(z) \in \mathrm{Lip}\,\ell$ on R then $w(T\hat{}(\theta))$ again corresponds to a bounded linear transformation of $\ell^{2,\alpha}$ to itself. It follows that the spectrum of T is R. Further it is shown that a necessary and sufficient condition for $\lambda$ to belong to the point spectrum of T is essentially that the set $\{\theta \mid T\hat{}(\theta) = \lambda\}$ be of positive $1+2\alpha$ capacity. (Received September 9, 1957.)

538-49. R. C. MacCamy: The principle of Babinet.

The classical "Principle of Babinet" in acoustic and electromagnetic theory states that the problems of diffraction by an aperture, S, in a screen y = 0, and by a screen occupying the same position S in the plane y = 0, are equivalent, i.e., solution of one yields simultaneously the solution of the other. H. Rubin (Comm. Pure Appl. Math. vol. 7 (1954) ) showed that a kind of Babinet Principle holds also in the diffraction of surface water waves. This paper extends the principle to more general boundary problems for elliptic differential equations in two variables, when data is given on a line say y = 0. It is shown that the boundary problem, find u(x,y) such that $u_{xx} + u_{yy} = 0$, in $y < 0$, $L(\partial/\partial x, \partial/\partial y)u = 0$ on $y = 0$ $|x| > a$, $M(\partial/\partial x, \partial/\partial y)u = g(x)$ on $y = 0$, $|x| < a$, where g(x) is given and L and M are polynomials is equivalent to a second boundary problem for harmonic functions with the condition $v_y = 0$ on $y = 0$, $|x| > a$. The latter problem is well adapted to solution by integral equations. Applications in the water wave problem and in the problem of the "oblique derivative" are given. (Received September 9, 1957.)

538-50. E. C. Dade and Karl Goldberg: Incidence algebras.

An incidence algebra is an algebra with a basis of 0,1 matrices $\{A_p\}$ whose sum is the matrix with 1 in every position. Given a permutation group of degree n an incidence algebra of matrices of order n can be constructed by using the group to partition the set of ordered pairs $\{(i,j): i,j = 1,2,\ldots,n\}$ into equivalence sets whose incidence matrices $\{A_p\}$ form the required basis. The equivalence is: $(i,j) \sim (i',j')$ if and only if there is an element of the group taking i into i'

and j into j'. The incidence matrix of an equivalence set has 1 in the i,j position if (i,j) is in the set and 0 otherwise. With this method it is shown that a sufficient condition for the construction of a Hadamard matrix of order 4k is that there exist a transitive permutation group of odd order and degree 4k-1 whose subgroups leaving one element fixed separate the other 4k-2 elements into exactly two transitivity sets. The paper also includes some theorems on general incidence algebras and an application to a problem in graph theory (too complex to be stated here) for which the solution is the set of structure constants for the $\{A_p\}$ of the relevant incidence algebra. (Received September 9, 1957.)

538-51. Sadayuki Yamamuro: On Nakano spaces.

Let R be a Nakano space (cf. Hidegoro Nakano, Modulared Semi-ordered Linear Spaces, 1950) with monotone complete, simple and finite modular m and its conjugate modular $\bar{m}$ be also simple and finite. The following theorem is proved: Let $\bar{a}_\nu \in \bar{R}$ ($\nu = 1,2,\ldots$), where $\bar{R}$ is the conjugate space of R. If $\sum_{\nu=1}^{\infty} \bar{a}_\nu(a_\nu) < +\infty$ for every sequence of elements $a_\nu$ ($\nu = 1,2,\ldots$) with $\sum_{\nu=1}^{\infty} m(a_\nu) < +\infty$, then $\sum_{\nu=1}^{\infty} \bar{m}(\alpha \bar{a}_\nu) < +\infty$ for some $\alpha > 0$. By this theorem, a sequence space representation of R can be obtained and its properties are discussed. (Received September 9, 1957.)

538-52. Trevor Evans and P. B. Schwartz: On Slupecki T-functions.

A many-valued logic is functionally-complete if, for any function on its set of truth-values, there is a formula of the logic whose truth table is this function. The Lukasiewicz-Tarski is not functionally complete but the addition of a new connective T, due to Slupecki, such that T(p) has truth-value 2 for all truth-values assigned to p, makes the logic functionally complete. Let $T_i$ be a connective such that $T_i(p)$ has truth-value i for all truth-values assigned to p and let $\mathcal{L}_i$ be the m-valued Lukasiewicz-Tarski logic with $T_i$ added. It is shown that $\mathcal{L}_i$ is functionally complete if and only if m-1 and $i_0$-1 are mutually prime where $i_0$ is the minimum of i and m-i+1. The proof depends on the construction of certain formulas in the Lukasiewicz-Tarski logic and on the Euclidean algorithm applied to m-1 and $i_0$-1. (Received September 10. 1957.)

538-53. V. L. Klee, Jr. and R. G. Long: On a method of mapping due to Kadeč and Bernstein.

The following important problem dates back to Frechet's book of 1928

and Banach's book of 1932: (I) Are all infinite-dimensional Banach spaces homeomorphic? In addition, a special case of the following was raised by Frechet: (II) Are all $\aleph_0$-dimensional normed linear spaces homeomorphic? Until recently, no significant progress was made on these problems. But in Doklady Akad. Nauk SSSR vol. 92 (1953) pp. 465-468, Kadeč has introduced a method of mapping in terms of the deviation-sequences of S. Bernstein (C.R. Acad. Sci. Paris vol. 206 (1938) pp. 1520-1523); this has enabled him to obtain a homeomorphism between $(c_0)$ and $(l^2)$, and looks promising as a route to affirmative solution of (I). (By a second method, more or less dual to the first, Kadeč has proved that all uniformly convex infinite-dimensional separable Banach spaces are homeomorphic. However, this second method does not seem amenable to useful extension.) The present paper supplies a careful exposition of the first method of Kadeč, motivated by the long standing of the problem and the fact that the arguments of Kadeč and Bernstein neglect several important points. There is obtained incidentally an affirmative solution of (II). (Received September 10, 1957.)

538-54. Herbert Federer: A note on the Gauss-Green theorem.

Recently De Giorgi (Ricerche di Matematica, vol. 4, 1955) as well as Fleming and Young (Rend. Circ. Mat. Palermo, ser. 2, vol. 5, 1956) have obtained interesting results concerning sets A in n-space which are measurable with respect to Lebesgue measure L and for which there exist locally finite Borel measures $\psi_1,\ldots,\psi_n$ such that $\int D_i f dL = \int f d\psi_i$ for $i = 1,\ldots,n$ whenever f is a sufficiently smooth function vanishing at infinity. In the Schwartz language of distributions this means that the partial derivatives of the characteristic function of A are measures. (Such distributions were characterized by Federer, Bull. Amer. Math. Soc. Abstract 60-4-407 and Krickeberg, Bull. Amer. Math. Soc. Abstract 63-4-437). On the other hand Federer studied (Trans. Amer. Math. Soc. vol. 58 (1945); vol. 59 (1946); vol. 62 (1947) ) the special case in which the n-1 dimensional Hausdorff measure H (Boundary A) is finite. In this study he defined the exterior normal $\nu(A,x)$, a unit or null vector, at any point x in n-space, in terms of geometric properties of A. It is now proved that in the general case the measures $\psi_i$ are related to the exterior normal $\nu$ by the formulae $\psi_i(B) = \int_B \nu_i(A,x) dHx$ for every Borel set B. The proof uses the results of De Giorgi and a new lemma stating that the total variation of the vector-valued measure defined by $\psi_1,\ldots,\psi_n$ has positive n-1 dimensional upper

density at x whenever $\nu(A,x)$ is not null. (Received September 10, 1957.)

538-55. C. W. Kohls: Prime ideals in rings of continuous functions. Preliminary Report.

The prime ideal structure of the ring C(X) of all continuous real-valued functions on a completely regular Hausdorff space is investigated (some background may be found in Bull. Amer. Math. Soc. Abstracts 63-2-171 and 63-2-172). (1) Each prime ideal of C(X) contains a prime $\mathfrak{Z}$-ideal that is a minimal prime ideal. (2) Let p be a nonisolated $G_\delta$-point of X, and let $\phi$ denote the extension of the identity mapping on $X \sim \{p\}$ into X to the largest subspace of $\beta(X \sim \{p\})$ to which it is extendible as a continuous function into X. A Z-filter on X corresponds to a (prime) $\mathfrak{Z}$-ideal that is maximal in the set of $\mathfrak{Z}$-ideals contained properly in $M_p$ iff it is the collection of closures in X of sets in a Z-ultrafilter on $X \sim \{p\}$ converging in $\beta(X \sim \{p\})$ to a point of $\phi^{-1}(\{p\})$. (3) Let I and J be prime $\mathfrak{Z}$-ideals contained in $M_p$, and such that Z(I) and Z(J) are contained in Z-filters on X corresponding to distinct Z-ultrafilters on $X \sim \{p\}$. Then the chain of prime ideals of C(X) containing I and the chain of prime ideals of C(X) containing J have only the element $M_p$ in common. (This work was supported in part by the National Science Foundation.) (Received September 11, 1957.)

538-56. J. F. Nash: The decrease of fundamental solutions of parabolic equations with distance from the initial singularity.

In the paper "Parabolic Equations" of the August 1957 Proc. Nat. Acad. Sci. a moment bound, $\int_r T dv \leq kt^{1/2}$, is derived for fundamental solutions of the heat equation $\nabla \cdot [C \cdot \nabla T] = \dot{T}$ where C is variable but has eigenvalues between positive bounds. From this one can derive a bound on the size of the fundamental solution T in terms of the distance r from the initial singularity: $T \leq k_1 \exp[-k_2 (r/kt^{1/2})^{k_3}]$ provided $r \geq k_4 t^{1/2}$. All constants depend only on the dimension of the space and the bounds on the eigenvalues of C. (Received September 11, 1957.)

538-57. Seymour Schuster: Pencils of null systems

Given a complete pentagon $A_0A_1A_2A_3A_4$ in complex projective space of three dimensions, a null system is determined by the correlation $A_i \leftrightarrow A_{i-1}A_iA_{i+1}$, where the subscripts are taken modulo 5. A family of $\infty^1$ null systems, called a pencil of null systems, is defined to be those which arise from

pentagons $A_0A_1A_2A_3A_4$, where one plane, say $A_0A_1A_2$, varies in an axial pencil about line $A_1A_2$. The self-polar lines of each null system form a linear complex. Thus, each pencil of null systems yields a family of $\infty^1$ linear complexes, called a pencil of linear complexes. Examining the set of those lines which are self-polar for all null systems of the pencil, viz. the lines common to all linear complexes of the pencil, it is seen that they form a linear congruence. A classification of pencils of null systems yields three essentially different types: (i) the general case, wherein the four fixed points of the pentagon are not coplanar; (ii) the pencil whose defining pentagon degenerates by having three consecutive points collinear; and (iii) the pencil defined by a pentagon, the four fixed points of which are coplanar (with no three collinear). These three cases give rise to the three different types of linear congruences. They are the general, parabolic and degenerate congruences, respectively. (Received September 11, 1957.)

538-58. Harold Widom: An extremum problem.

Let $f(\theta) \geq 0$ and $\in L(-\pi,\pi)$; let L be a linear functional on the space of polynomials in z and set $\alpha_k = L(z^k)$, $k = 0,1,\cdots$. Define $m(f,L) = \inf\{(1/2\pi)\int_{-\pi}^{\pi} |P(e^{i\theta})|^2 f(\theta)d\theta : L(P) = 1\}$. THEOREM I: If $\sum |\alpha_k|^2 = \infty$ and $f \in L_2(-\pi,\pi)$ then $m(f,L) = 0$. In case $\sum |\alpha_k|^2 < \infty$, the function $g(z) = \sum_0^\infty \alpha_k z^k$ is in $H_2$ of the unit circle. We define $F_+(z) = \exp[(1/2\pi)\int_{-\pi}^{\pi} (ze^{-i\theta} - 1)^{-1} \log f(\theta)d\theta]$ for $|z| < 1$ and $F_-(z)$ by exactly the same formula for $|z| > 1$. THEOREM II: If $\sum |\alpha_k|^2 < \infty$ and $f(\theta)$ is (essentially) bounded and bounded away from zero, then $m(f,L)^{-1} = G(f)\sum_0^\infty |(1/2\pi i)\int_{|z|=1} F_+(z)g(z^{-1})z^{k-1}\,dt|^2$ where $G(f) = F_+(0)^{-1} = \exp((1/2\pi)\int_{-\pi}^{\pi} \log f(\theta)d\theta)$. THEOREM III: If $\limsup |a_n|^{1/n} = r_0 < 1$, and $\log f(\theta) \in L_1(-\pi,\pi)$ then $m(f,L)^{-1} = G(f)\sum_0^\infty |(1/2\pi i)\int_{|z|=r} F_+(z)g(z^{-1})z^{k-1}\,dz|^2$ where $r_0 < r < 1$. The above generalizes a theorem of Szegö [Math. Zeit. vol. 6 (1920)] which gives the result for $L(P) = P(\alpha)$, where $\alpha$ is complex and $|\alpha| < 1$. (Received September 11, 1957.)

538-59. P. M. Anselone: Integral equations of the Schwarzchild-Milne type.

The Schwarzchild-Milne integral equation of radiative transfer theory is $J(x) = (\Lambda J)(x)$, where $\Lambda$ is defined by $(\Lambda f)(x) = \int_0^\infty E_1(x-y)f(y)dy$, $x \geq 0$, and $E_1(t)$ is the exponential integral of first order. This equation is generalized by replacing the kernel $E_1(t)$ by any function $H_1(t)$ which satisfies the conditions $H_1(t) \geq 0$, $\int_{-\infty}^{\infty} H_1(t)dt = 1$, $\int_{-\infty}^{\infty} H_1(t)tdt = 0$ and $H_2(t) \leq CH_1(t)$ for some positive constant C, where $H_2(t) \int_t^\infty H_1(s)ds$ for $t \geq 0$ and $H_2(t) = \int_{-\infty}^{t} H_1(s)ds$ for $t < 0$.

The substitution of $J(x) = x + \phi(x)$ into $J = \Lambda J$ yields $\phi - \Lambda\phi = H_2$. The Neumann series, $q(x) = \sum(\Lambda^n H_2)(x)$, converges uniformly on each finite x-interval and satisfies $q - \Lambda q = H_2$. The complete solution of the problem for J(x) defined by $J = \Lambda J$, $J \geq 0$, $J \not\equiv 0$ is given by $J(x) = b[x + q(x)]$, where b is an arbitrary positive constant. Various properties of q(x) are derived, including $0 < q(x) \leq C$. (Received September 11,1957.)

538-60. Joachim Nitsche and J. C. C. Nitsche: A criterion for the existence of nonlinear entire solutions of elliptic equations.

Let (*) $a(p,q)\cdot r + 2b(p,q)\cdot s + c(p,q)\cdot t = 0$ be a quasilinear elliptic differential equation arising from a variational problem $\delta\iint F(p^2 + q^2)dxdy = 0$. Here the function $F(w)$, $w = p^2 + q^2$, is assumed to possess, for all non-negative values of w, a Hölder-continuous third derivative. Put $\lambda(w) = 2F''(w)/F'(w)$. Equation (*) is elliptic if $F' \neq 0$, say $F'(w) > 0$ and $1 + w\lambda(w) > 0$. Authors prove the following theorem: Under the assumptions formulated above there exist nonlinear entire solutions if the integral $\int^{\infty}(1 + w\lambda(w))/(2 + w\lambda(w))(dw/w)$ diverges. This criterion is explicit (for another criterion see L. Bers, J. Rat. Mech. Anal. vol. 3, Theorem II and p. 785). The proof is based on (i) the introduction of "normal functions" (J. Nitsche, Math. Nachr. vol. 7),(ii) a mapping theorem by L. Bers (Comm. Pure Appl. Math. vol. 6), and (iii) a lemma by M. A. Lavrent'ev on quasiconformal mappings (Rec. Math. vol. 42). (Received September 11, 1957.)

538-61. R. S. Ledley: The functional equation in finite Boolean algebras.

Functional equations arise in the logical design of digital computers. For example, how should already built digital circuits be logically connected so that the resulting combination will produce a predetermined Boolean function. Systematic digital computational methods have been developed to solve such problems. In mathematical terms the problem is: given a Boolean function $E(A_1,\ldots, A_i, X_1,\ldots,X_k)$, and the set of functions $f_1(A_1,\ldots,A_i),\ldots, f_j(A_1,\ldots,A_i)$, find all functions $F(f_1,\ldots,f_j,X_1,\ldots,X_k)$ such that $F \rightarrow E$ (the antecedence problem), or such that $E \rightarrow F$ (the consequence problem). The method is embodied in the Boolean matrix equations: $(R_{ji}) \otimes (E_{ik}) = (F_{jk})$ for antecedence solutions F, and $(R_{ji}) \otimes (E_{ik}) = (F_{jk})$ for consequence solutions F, where $\otimes$ is logical matrix multiplication and the matrices are formed and interpreted as in Proc. Nat. Acad. Sci. vol. 41, no. 7, pp. 498-511. It turns out that there is a l.u.b. for

the set of antecedence solutions and a g.l.b. for the set of consequence solutions. (Received September 12, 1957.)

538-62. R. T. Ives: Uniform shrinkability in topological vector spaces.

A neighborhood A of the origin is uniformly shrinkable iff for each $t \in ]0, 1[$, there is a neighborhood U of the origin such that $tA + U \subset A$. This is proved equivalent to the condition: for each $t \in ]0, 1[$, there is a neighborhood W of the origin such that line segments joining points in W are contained in A. The $L^p$ spaces ($0 < p < 1$) are trivially locally uniformly shrinkable. It is proved that certain spaces which are not locally bounded are also not locally uniformly shrinkable. One of these is the space of all measurable functions on the real line with convergence in measure. The general relationship to local boundedness is unknown. (Received September 12, 1957.)

538-63. Sigurdur Helgason: On a theorem of Sidon.

Let G be the circle group, C(G) the Banach space of continuous functions on G equipped with the uniform norm, and let A(G) be the subspace of C(G) consisting of functions that have absolutely convergent Fourier series. Let T be a bounded operator on C(G) which is invariant under rotations. Sidon has in the paper "Ein Satz über die Fourierschen Reihen stetiger Funktionen" Math. Zeit. vol. 34 (1932) pp. 485-486, proved the following theorem: If T leaves the subspace A(G) invariant, T has the form $Tf = F * f$ (convolution product) where $F \in L^2(G)$. In the present paper Sidon's theorem is extended to all compact groups, commutative or not. The proof is obtained by using the results on spectrally continuous operators proved by the author in Trans. Amer. Math. Soc., 1957. (Received September 12, 1957.)

538-64. Sigurdur Helgason: Differential operators on homogeneous Riemannian spaces.

Let G be a connected Lie group, K a compact subgroup. The homogeneous Riemannian space G/K of cosets gK carries a metric invariant under the action of G. Let $\pi$ be the natural projection of G onto G/K and T the tangent space to G/K at $\pi(e)$. The system D(G/K) of differential operators on G/K that are invariant under G can then be put in a 1-1 linear degree preserving correspondence with the set of polynomials on T that are invariant under the linear group ad(K). In particular, if ad(K) is transitive on the directions in T, D(G/K)

consists of all polynomials in the Laplace-Beltrami operator $\Delta$ on G/K. For each $x \in G$ we consider the operator $M^x$ given by $(M^x f)(\pi(g)) = \int_k \tilde{f}(gkx)dk$ where f is a continuous function on G/K, $\tilde{f} = f \circ \pi$ and dk is the normalized Haar measure on K. $M^x$ is a generalisation of the spherical mean value in Euclidean spaces (see R. Godement, C. R. Acad. Sci. (1952) pp. 2137-2139). One can now prove a formula that gives the mean value operator $M^x$ in terms of the differential operators in D(G/K). In case G/K is the euclidean n-space $R^n$ the formula for $M^x$ is the known expansion $M^r = \Gamma(n/2) \sum_0^\infty (r/2)^{2m} [m! \Gamma(m + n/2)]^{-1} \Delta^m$ Various mean value theorems for differential equations in $R^n$ involving the Laplacian can thus be extended to homogeneous Riemannian spaces. (Received September 12, 1957.)

538-65. Sigurdur Helgason: Mean value theorems on symmetric spaces.

Keeping the notations from the preceding abstract, (Differential operators on homogeneous Riemannian spaces) we consider now the special case where G/K is a simply connected symmetric Riemannian space of rank 1. Let u be a harmonic function on a domain of G/K, that is a function satisfying the equation $\Delta u = 0$. Then the mean value of u is constant on a family of confocal ellipsoids in G/K. (If $E_1$ and $E_2$ are confocal ellipsoids with center q in the tangent space $T_q$ to G/K in q then $\exp_q E_1$ and $\exp_q E_2$ are called confocal ellipsoids in G/K, $\exp_q$ denoting the usual mapping of $T_q$ into G/K which maps straight lines onto geodesics). Next let u(x,y) be a $C^2$-function on $G/K \times G/K$ satisfying the equation $\Delta_x u(x,y) = \Delta_y u(x,y)$. Then for each $x_0, y_0 \in G/K$, $r > 0$ we have $\int_{S^r(x_0)} u(x,y_0)dw(x) = \int_{S^r(y_0)} u(x_0,y)dw(y)$ where dw is the surface element on the geodesic sphere $S^r$ of radius r. The proof breaks up into two cases. The case G/K compact is settled by Fourier analysis. In the noncompact case one uses E. Cartan's theorem that G/K has everywhere nonpositive curvature and implications this theorem has for the geodesics. Finally, if f is a $C^2$-function on G/K the function $F(y,x) = (M^x f)(y)$ satisfies the Darboux equation $\Delta_x F(y,x) = \Delta_y F(y,x)$. In the case of a euclidean space, these theorems are due to Asgeirsson (Math. Ann. vol. 113 (1936) pp. 321-346). (Received September 12, 1957.)

538-66. Valdemars Punga: On the Monte Carlo solution of simultaneous linear equations.

The aim of this paper is to demonstrate the simplicity of Monte Carlo method devised by J. von Neumann and S. M. Ulam and presented by G. E. Forsythe and R. A. Leibler (Math. Tables and Aids to Computation vol. 4, 1950, pp. 127-129) for matrix inversion, when applied to the solution of some simultaneous linear equations. A simple interpretation is given to corresponding stochastic process which permits one using tables of random numbers to obtain in a curious Monte Carlo way an approximate solution for a system of three or four linear equations, without digital computer, in a reasonable time. (Received September 12, 1957.)

538-67. J. C. E. Dekker: The factorial function for isols.

For notations and terminology see Abstract 63-5-610. We write $B/A$ if $B \cdot X = A$ for some $X$ in $\Lambda$. If $B/A$ and $B \neq 0$, the unique element $X$ such that $B \cdot X = A$ is denoted by $A/B$. An isol is called cosimple (cohypersimple) if it can be represented by the complement of a simple (resp. hypersimple) set. R. S. Tennenbaum suggested (oral communication) a definition for the factorial function in $\Lambda$ using permutations of $\epsilon$ which map almost all numbers onto themselves. Using this definition the author proves: (1) $(X + 1)! = (X + 1) \cdot X!$, (2) $X! \cdot Y!/(X + Y)!$, (3) if $X$ is cosimple (cohypersimple) $X!$ is also cosimple (resp. cohypersimple). In view of (2) we see: (4) $B \leqq A$ implies $B! \cdot (A - B)!/A!$ This suggests the definition: $C(A,B) = A!/B! \cdot (A - B)!$ for $B \leqq A$. One can now prove purely algebraically: (5) $C(A + 1, B + 1) = C(A,B) + C(A,B + 1)$ for $B + 1 \leqq A$. Myhill observed (oral communication) that (3) and (4) imply: (6) there exists a collection of cohypersimple isols which has order type $\eta$ when ordered by $\prec$. (Received September 12, 1957.)

538-68. Harald Holmann: Complex manifolds with a group of holomorphic automorphisms.

If an $n$-dimensional $(p_1,\ldots,p_n)$-domain is defined by the property that it admits the automorphisms $A(\theta)$: $z_\nu^* = \exp(ip_\nu \theta)\, z_\nu$, $\theta$ real, $p_\nu$ integers, $\nu = 1,..,n$, Cartan's Mapping Theorem, J. Math. Pures Appl. vol. 10 (1931) pp. 1-114, can be generalized as follows: Let $M_n$ be a complex manifold, which admits an 1-dimensional compact Lie group G (not necessarily with a fixpoint) of holomorphic automorphisms $F(\theta)$, $0 \leqq \theta \leqq 2\pi$, then each point P of $M_n$ has a

neighborhood U(P) invariant with respect to the transformations of G, such that there exists a 1-1 holomorphic mapping T of U(P) onto a $(p_1,\dots,p_n)$-domain, which is compatible with the automorphisms $A(\theta)$ and $F(\theta)$, that means: $TF(\theta) = A(\theta)T$, if $F(\theta)$ is given in the canonical form. There are examples to prove that a global analogon of Cartan's Mapping Theorem does not exist in general. In the case of Stein manifolds, however, if the group G has a fixpoint, under certain conditions the whole manifold can be mapped onto a $(p_1,\dots,p_n)$-manifold, which locally has the structure of a $(p_1,\dots,p_n)$-domain and always is contained in a canonically associated vector-bundle. Furthermore a necessary and sufficient cohomology condition exists, under which a Stein manifold is holomorphically equivalent to a $(p_1,\dots,p_n)$-domain. (Received September 12, 1957.)

538-69. Saul Kravetz: On the local geometry of Teichmueller spaces. I. Preliminary report.

Having studied [Bull. Amer. Math. Soc. vol. 59 (1953) p. 541; vol. 63 (1957) p. 235] the global geometry of Teichmueller spaces, the author initiates the study of their local geometry. It is shown that any Teichmueller space (with the Teichmueller metric) is a Finsler space (and an expression for the Finsler metric is deduced) by proving a slightly stronger version of Busemann's condition $\Delta$ [Metric methods in Finsler spaces and in the foundations of geometry, p. 48]. (Received September 12, 1957.)

538-70. Robert Osserman: Relations between mean curvature and conformal type.

If a surface S is given by $z = f(x,y)$ over some region of the plane, and if the z-coordinate of each point of S is considered as a function on the surface, then a computation shows that $\Delta z = 2H/W$ where $\Delta$ is the second Beltrami operator, H the mean curvature, and $W^2 = 1 + z_x^2 + z_y^2$. From this we deduce immediately: 1. S is a minimal surface if and only if z is harmonic in local isothermic parameters; 2. S is of hyperbolic type (as a Riemann surface) if z is nonconstant, $H \geqq 0$ and $z \leqq M$ everywhere. It may be noted that if f(x,y) is defined over the whole x,y-plane, then it cannot satisfy these last two conditions unless it is constant. (Received September 12, 1957.)

538-71. G. D. Findlay and Joachim Lambek: Rational extensions of modules.

Given a ring R, we call a right R-module $B_R$ a rational extension of a submodule $A_R$ if and only if every homomorphism of $A_R$ into $B_R$ can be uniquely extended to a maximal partial endomorphism of $B_R$. $A_R$ possesses a maximal rational extension $\bar{A}_R$, unique up to isomorphism. Let $E_R$ be a maximal essential extension of $A_R$. Then $\bar{A}_R$ consists of all elements of $E_R$ which are annihilated by all endomorphisms of $E_R$ which annihilate $A_R$. The maximal rational extension of $R_R$ is a ring and coincides with the quotient ring or R as defined by Utumi if R is left faithful (Y. Utumi, On quotient rings, Osaka Math. Journal vol. 8 (1956) pp. 1-18). There are applications to multiplicative ideal theory. (Received September 12, 1957.)

538-72. V. J. Mizel: A boundary layer result for an N-dimensional linear elliptic equation.

Recently, a technique developed by S. L. Kamenomostkaya has been applied by both D. G. Aronson and the author to two-dimensional boundary layer problems involving linear equations with a maximum principle. Here that technique is applied to extend a result of N. Levinson's for the two-dimensional linear elliptic equation to the n-dimensional case. We consider the uniformly elliptic equation (Einstein's summation convention): $\epsilon a_{ij}(x)u_{x_ix_j} + a_i(x)u_{x^i} + c(x)u = d(x), c(x) < 0$, $a_{ij} = a_{ji}$ on a region D, both equation and region being sufficiently well-behaved so that for each $\epsilon > 0$ there exists a solution to the first boundary value problem $u(x, \epsilon) = \bar{u}(x)$ on D. By direct comparison with certain boundary layer terms it is shown that in the interior of any "characteristic tube" (defined for the $\epsilon = 0$ case) the solution $u(x,\epsilon)$ approaches as $\epsilon \to +0$ that solution U(x) of: $a_i(x)U_{x^i} + c(x)U = d(x)$ whose initial value coincides with $\bar{u}(x)$ at one of the bases of the tube. In particular, it is shown that in such a tube $u(x,\epsilon) = U(x) + Z(x,\epsilon) + w(x,\epsilon)$ where $Z(x,\epsilon)$ has the form $h(x)e^{-g(x)/\epsilon}$ in a neighborhood of one base of the tube ($g = 0$ on that base, $g > 0$ off that base) and elsewhere in the tube is uniformly $O(e^{-\delta_1/\epsilon})$ for a fixed $\delta_1 > 0$, and where $w(x,\epsilon)$ is uniformly $O(\epsilon^{1/2})$ and vanishes on the bases of the tube. (Received September 12, 1957.)

538-73. Naoki Kimura: On some special idempotent semigroups (I).

An idempotent semigroup is called regular (left regular) if abaca = abca (aba = ab) is satisfied for any a, b, c. Then a regular idempotent semigroup can

be uniquely expressed as a special kind of product of a left regular idempotent semigroup and a right regular idempotent semigroup. The representation theorem of these idempotent semigroups is also given. (Received September 13, 1957.)

538-74. P. E. Zadunaisky: On the group iteration and group overrelaxation methods of solving a system of linear equations.

Group iteration can be considered a generalisation of Seidel's method; in this paper some results of Collatz (Z. F. Angew. Math. Mech. vol. 22 (1942) p. 360) and Geiringer (Contr. to Appl. Mech., Reissner, 1948), concerning convergence and an estimation of the error in Seidel's method, are extended to group iteration, assuming that the matrix of the coefficients is irreducible and that the largest row sum, taken as a norm of the matrix, is $\leq 1$. The case of an "almost reducible" matrix is also considered. Comparing both methods it is shown that group iteration converges faster than Seidel's method even when the matrix is not symmetric. Further it is shown that this advantage can be preserved when overrelaxation ( D. Young, Trans. Amer. Math. Soc. vol. 76 (1954)) is applied to group iteration, rather than to Seidel's method, provided the conditions assumed above are satisfied. These methods have been tested on an IBM 650 Computing Machine at Watson Laboratory; for a system of equations where the matrix was non symmetric the predicted improvements on the speed of convergence were found to agree reasonably well with the numerical results. (Received September 13, 1957.)

538-75. James Serrin: The Harnack inequality for elliptic partial differential equations in more than two independent variables.

We consider the second order partial differential equation of elliptic type $Lu = a_{ik}(x)u_{ik} + b_i(x)u_i + c(x)u = 0$, where $x = (x_1,\ldots,x_n)$ and standard notational conventions are used. It is assumed that the coefficients are bounded ($< M$) and that L is uniformly elliptic, i.e. there exists a constant $\lambda > 0$ such that, for all points x and all real n-tuples $(\xi_1,\ldots,\xi_n)$, $\lambda^{-1}|\xi|^2 \leq a_{ik}\xi_i\xi_k \leq \lambda|\xi|^2$. Then the following theorem holds (cf. Journal d'Analyse Mathématique vol. 4 (1956) p. 299): Let u(x) be a positive solution of the equation $Lu = 0$ in the unit circle, and suppose that the coefficients $a_{ik}$ are Hölder continuous. Then there exists a constant $K > 0$, depending only on $\lambda$, M, and on the Hölder moduli of the $a_{ik}$, such that $u(x) \leq Ku(0)$ whenever $|x| < 1/3$. By a refinement of the proof of the

above result it is now shown that the same conclusion holds when the $a_{ik}$ are merely assumed to be continuous, the constant K in this case depending only on $\lambda$, M, and the modulus of continuity of the $a_{ik}$. This result enables one to prove the following Liouville theorem. Let u(x) be a positive solution over $E^n$ of the uniformly elliptic equation $a_{ik} u_{ik} = 0$, where the coefficients $a_{ik}$ are continuous and tend to a limit as $x \to \infty$. Then $u \equiv$ constant. (Received September 16, 1957.)

538-76. James Serrin: On solutions of elliptic partial differential equations which are positive near an isolated singular point, I.

We consider the differential equation $Lu = 0$ defined in the preceding abstract, with $n > 2$. The singular point will be taken at the origin. It is assumed that the $a_{ik}$ are continuous in some sphere $0 \leqq r \leqq r_0$, and, for simplicity, we also suppose that $c \leqq 0$ and that the coefficients of L are continuously differentiable in the punctured sphere S: $0 < r \leqq r_0$. Then we prove (1) there exists a solution v(x) of $Lu = 0$ in S with the property that, for every $\delta > 0$, the inequality $r^{2-n+\delta} < v < r^{2-n-\delta}$ holds if r is suitably small, and (2) every solution of $Lu = 0$ in S which is positive in the neighborhood of the origin has the form $u = av + w$, where $a = \text{const.} \geqq 0$ and w is of class $C^1$ in $0 \leqq r \leqq r_0$. It is noted that v need not be of the order $r^{2-n}$ as $r \to 0$, this behavior occuring in general only when the $a_{ik}$ are Hölder continuous at the origin (cf. Journal d'Analyse Mathématique vol. 4 (1956) p. 336). The above results remain true when the differentiability assumption on the coefficients is dropped, provided a "solution" now means a $C^1$ solution in the sense of Morrey (Annals of Mathematics Studies, no. 33, pp. 101-159). Proofs are based on the Harnack inequality (preceding abstract) and on various comparison arguments. (Received September 16, 1957.)

538-77. James Serrin: On solutions of elliptic partial differential equations which are positive near an isolated singular point, II.

Results similar to those of the preceding abstract also hold when the singular point is at infinity, that is, when the solution is defined in some exterior region $S^*$: $r_0 \leqq r < \infty$. Consider in particular the uniformly elliptic equation $Lu = a_{ik} u_{ik} = 0$, where the coefficients $a_{ik}$ are continuously differentiable in $S^*$ and tend to limits as $x \to \infty$. Then we prove (1) there exists a solution $v^*(x)$ of $Lu = 0$ in S with the property that, for every $\delta > 0$, the inequality $r^{2-n-\delta} < v^* < r^{2-n+\delta}$ holds if r is suitably large, and (2) every solution of

$Lu = 0$ in $S^*$ which is positive for large r has the form $u = a + bv^* + w$, where a and b are constants and $w = o(v^*)$ as $r \to \infty$, (in particular, u must tend to a finite limit as $x \to \infty$). Furthermore, w satisfies an extended form of the maximum principle: in any sphere $r_1 \leq r < \infty$ its maximum (and minimum) is attained on the boundary $r = r_1$. This theorem is an extension of certain results of D. Gilbarg and the author (Journal d'Analyse Mathématique vol. 4 (1956) and of N. G. Meyers (Bull. Amer. Math. Soc. vol. 63 (1957) p. 273). (Received September 16, 1957.)

538-78. Morris Newman and Irving Reiner: Inclusion theorems for congruence subgroups.

Let $G_t$ be the multiplicative group of $t \times t$ rational integral matrices of determinant 1. For a fixed partition $t = r + s$, r,s positive integers and for a positive integer n define the subgroup $G_r(n)$ as the totality of matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ of $G_t$ such that A is an $r \times r$ matrix, D an $s \times s$ matrix, and $C \equiv 0 \pmod{n}$. It is shown that if H is a subgroup of $G_t$ containing $G_r(n)$ then $H = G_t(n_1)$ for some divisor $n_1$ of n. The analogous theorem is proved for the symplectic modular group. In addition the theorem is generalized to intersections of groups $G_r(n)$, and to congruence subgroups with two coprime moduli. (Received September 23, 1957.)

538-79. Gert Sabidussi: The structure of graphs.

The following structure theorem for finite connected graphs is proved. Given a finite connected graph X let $X_0$ be a maximal connected subgraph of X such that no two distinct edges of $X_0$ are similar in X, then $X = (X_{G(X),H} \circ Y)/G_0$ where G(X) is the automorphism group of X, $G_0$ is that subgroup of G(X) which leaves $X_0$ invariant, and $H = \{\psi \in G(X) \mid V(X_0) \cap V(\psi X_0) \neq \square\}$ is a set of generators of G(X). $X_{G(X),H}$ is the color-group of G(X) with respect to H, Y is the maximal subgraph of X with $V(Y) = V(X_0) \cup \bigcup_{\psi \in H} V(\psi X_0)$, and $X_{G(X),H} \circ Y$ is obtained from $X_{G(X),H}$ by replacing each vertex $\phi$ of $X_{G(X),H}$ by a copy $Y_\phi$ of Y and identifying the vertices of $\eta_\phi P_\psi$ with the vertices of $\eta_{\phi\psi} Q_\psi$, where $\eta_\phi$ is an isomorphism of Y onto $Y_\phi$, and $P_\psi = V(X_0) \cap V(\psi X_0)$, $Q_\psi = V(X_0) \cap V(\psi^{-1} X_0)$, $\psi \in H$. The symbol / denotes the operation of forming the quotient graph of $Z = X_{G(X),H} \circ Y$ with respect to the equivalence relation defined on V(Z) by $z \sim z'$ iff there exist $\phi$, $\phi' \in G(X)$ and $y \in V(Y)$ such that $z = \phi y, z' = \phi'$ and $\phi^{-1}\phi' \in G_0$. (Received September 23, 1957.)

539-1. Peter Henrici: A unified theory of propagated error and numerical stability for some methods of integrating ordinary differential equations.

Since Rutishauser's discovery (Z. angew. Math. Physik vol. 3 (1952) pp. 65-74) of weak numerical instability (our terminology) in some methods for solving ordinary differential equations the question has remained open how Rutishauser's results tie in with classical results on the propagation of the truncation error. A unified theory is now presented which covers both types of errors. At the same time Rutishauser's stability analysis is generalized from equations with constant coefficients to the general equation (i)$y' = f(x,y)$. For the integration of (1) by Simpson's rule there results the following: If the starting values are exact, the overall error satisfies (2) $\epsilon(x) = \eta(x)h^4 + [\gamma(x) + (-1)^n\omega(x)]h^5 + O(h^6)$. Here $x = x_0 + nh$, and $\eta(x) = (1/180)e^{G(x)}\int_{x_0}^{x} e^{-G(\xi)} f^{(4)}(\xi)d\xi$, $\gamma(x) = (1/360)f^{(4)}(x_0)e^{G(x)}$, $\omega(x) = -(1/360)f^{(4)}(x_0)e^{-(1/3)G(x)}$, where $f(x) = f(x,y(x))$, $G(x) = \int_{x_0}^{x} g(\xi)d\xi$, $g(x) = f_y(x,y(x))$, $y(x)$ being the true solution. If $G(x) < 0$, the error is dominated by the rapidly oscillating term involving $\omega(x)$. For $G(x) \geqq 0$ the oscillations are not conspicuous. By judicious choice of the starting values the $O(h^5)$-term in (2) can be made to disappear, but instability may still make itself felt through higher order terms. Similar results are given for $y_{n+1} = y_{n-1} + 2h\,f(x_n,y_n)$. (Sponsored by the Office of Ordnance Research.) (Received July 5, 1957.)

539-2. Peter Henrici: Error and numerical stability of two three-step methods for integrating $y' = f(x,y)$.

(a) An explicit asymmetric formula based on numerical differentiation. The formula (1) $y_{n+1} = y_{n-1} + 2h\,f(x_n,y_n)$ is known to be (weakly) unstable for $G(x) < 0$ (notation of preceding abstract). If (1) is replaced by (2)$y_{n+1} = (3/4)y_n + (1/4)y_{n-2} + (3h/2)f(x_n,y_n)$, there results a method of the same order, which however is strongly stable (all extraneous solutions decay hyperexponentially). The error of (2) as $h \to 0$ is twice the error of (1). (b) An implicit method based on Simpson's three-eights rule. Simpson's three-eights rule suggests the formula (3) $y_{n+1} = y_{n-2} + (3h/8)(f_{n-2} + 3f_{n-1} + 3f_n + f_{n+1})$. This is

weakly unstable for $G(x) < 0$. However, the extraneous solution (represented by $\omega(x)$ in the preceding abstract) grows like $e^{-(1/8)G(x)}$, that is, more slowly than for Simpson's ordinary rule. The dominating term of the overall error is 9/4 times the $\eta(x)h^4$ of the ordinary rule. (Sponsored by the Office of Ordnance Research.) (Received July 5, 1957.)

539-3. Peter Henrici: The numerical integration of certain ordinary differential equations with missing derivatives. Prelimary report.

The following formula is proposed for the step-by-step integration of the differential equation (1) $y^{(2n)} = f(x,y)$ (n a positive integer): (2) $\delta^{2n} y_i = h^{2n} \sum_{k=0}^{K} \sigma_{n,k} \delta^{2k} f(x_i, y_i)$ ($K \leq n$). Here $\sigma_{n,k} = n(n-k)^{-1} a_k^{(n-k)}$ $(n \neq k)$, $\sigma_{k,k} = \sum_{q=0}^{k} (q+1)\, a_{k-q}^{(-1)}\, a_q^{(1)}$, where $[2(\cosh z^{1/2} - 1)]^n = z^n \sum a_k^{(n)} z^k$. For $n = 1$, $K = 0,1,(2)$ reduces to familiar formulas. If the starting values are exact, the overall error of (2) satisfies (3) $\epsilon(x) = h^{2K+2} \eta(x) + O(h^{2K+3})$, where $\eta^{(2n)} = f_y(x,y(x))\, \eta - \sigma_{n,K+1} f^{(2K+2)}(x,y(x))$, $\eta^{(p)}(x_0) = 0$ $(p = 0,\ldots,2n-1)$, $y(x)$ being the true solution. By choosing the starting values judiciously (different from the exact values), $O(h^{2K+3})$ in (3) can be replaced by $O(h^{2K+4})$. Methods and results concerning error are extended to systems of equations of the form (1) and to equations of odd order. (Sponsored by the Office of Ordnance Research.) (Received July 5, 1957.)

539-4. Iwao Sugai: On unequal partitions of integers under certain imposed conditions.

For positive integers of i and n; $(i)^n$ is expressed as the sum of i distinct positive integers, such as $(i)^n = \sum_{m=1}^{m=i} a_{ni,m}$ where $i = 1,2,3,\ldots,n,\ldots$ . From these number elements an infinite "number-triangle" is formed. If $a_{ni,i}$, $(i = 1,2,3,\ldots,n,\ldots)$ and either $a_{ni,1}$ $(i = 1,2,3,\ldots,n,\ldots)$ or $a_{ni,m}$ $(m = 1,2,3,\ldots,n,\ldots)$ have zero n-th finite differences, the possible number of such "number-triangle" is $3 \cdot 2^{n-3}$ for $n \geq 3$. Thus under aforementioned conditions the integer $(i)^n$, $(n \geq 3)$ have $3 \cdot 2^{n-3}$ possible unequal partitions. Three major classifications of "number-triangles" are found. (Received August 5, 1957.)

539-5. R. K. Getoor: An analogue of Mercer's theorem.

The following theorem is proved. Theorem. Let X be a locally compact Hausdorff space and m a $\sigma$-finite Radon measure over X such that $L_2(X,m)$ is separable. Let $K(y,x)$ be a complex valued function defined on $X \times X$ which is

jointly measurable in (y,x) and satisfies the following conditions: (i) $K(y,x) = \overline{K(x,y)}$ (ii) $K(x,x)$ is continuous (iii) $K(\cdot,x) \in L_2(m)$ for each fixed $x \in X$ and the map $x \to K(\cdot,x)$ from X into $L_2(m)$ is continuous (iv) $\int\int K(y,x)f(y)\overline{f(x)}dm(y)dm(x) \geqq 0$ for all $f \in L_2(m)$ satisfying $\int |f(x)| \cdot \|K(\cdot,x)\| \, dm(x) < \infty$. Under these conditions there exist finite measures, $\mu_n$, with support contained in the non-negative reals and kernels $w_n(\lambda,x)$ jointly measurable in $\lambda \geqq 0$ and $x \in X$ such that $K(y,x) = \sum_1^\infty \int_0^\infty \lambda w_n(\lambda,y)\, \overline{w_n(\lambda,x)}\, d\mu_n(\lambda)$ a.e. on $X \times X$. Moreover, the sum converges uniformly on compact sets. The proof is based on the theory of generalized eigenfunction expansions as developed by Mautner, Proc. Nat. Acad. Sci. vol. 39 (1953); Garding, Lecture no. II, Institute for Fluid Dynamics and Appl. Math., Univ. of Maryland, 1954; and Bade and Schwartz, Proc. Nat. Acad. Sci. vol. 42 (1956). (Received August 21, 1957.)

539-6. V. L. Klee, Jr: Extremal structure of convex sets. II.

The paper is a collection of results, examples, and unsolved problems concerning the extremal structure of convex sets, dealing especially with exposed points in the sense of Straszewicz (Fund. Math. vol. 24 (1935) pp. 139-143), considered also by Milman (Doklady Akad. Nauk SSSR (N.S.) vol. 59 (1948) pp. 1045-1048). An exposed point of a set X in a topological linear space is a point p of X such that X is supported by a closed hyperplane which intersects X only at p. The principal result of the paper asserts that if C is a locally compact closed convex subset of a normed linear space and C contains no line, then ext $C \subset$ cl exp C and $C =$ cl conv (exp $C \cup$ rexp C), where ext C is the set of all extreme points of C, exp C is the set of all exposed points, and rexp C is the union of all exposed rays. (Compare with results of Extremal structure of convex sets. I., presented at the 1957 Summer Meeting.) Relationships between exposed points and extreme points are illuminated by several examples, and in particular there is constructed in $E^3$ a compact convex body C with boundary B such that $B \sim$ ext C, ext $C \sim$ exp C, and exp C are all dense in B. (Received August 26, 1957.)

539-7. F. L. Bauer and Evelyn Frank: Note on formal properties of certain continued fractions.

It is the purpose of this note to point out the connection between the work of Bauer (S.-B. Math.-Nat. Kl. Bayer. Akad. Wiss. (1956) pp. 163-203) and Frank (Trans. Amer. Math. Soc. vol. 81 (1956) pp. 453-476) on certain continued frac-

tions. A matrix approach similar to that of Bauer (loc. cit.) shows clearly that these are analogues of Stieltjes-type and Jacobi-type continued fractions. Examples are given by an expansion used for numerical purposes in Bauer (loc. cit.) which is closely related to the Euler expansion (Opera Omnia, Series Prima, vol. 8, pp. 362-390). This is obtained by a factorization of the Frobenius matrix. Special cases of these expansions coincide with certain cases of the hypergeometric continued fraction of Frank (loc. cit.). There is also a connection between the expansions derived in this note and the extended Schur continued fractions (cf. Frank (Proc. Amer. Math. Soc. vol. 3 (1952) pp. 921-937)). (Received September 3, 1957.)

539-8. Milton Lees and J. R. M. Radok: Explicit difference methods with high order accuracy for parabolic equations.

The prevalent use of the implicit method for the numerical solution of the simple diffusion equation is due to the unstable behavior for large forward time steps of the otherwise more convenient explicit method. A process of stabilization of the explicit method has been established which, in combination with the developed truncation error control, places the explicit method in a competitive position with respect to the implicit method. (Received September 20, 1957.)

539-9. A. M. Chak: A class of polynomials analogous to Lommel polynomials.

Mitra-Sharma (Bull. Cal. Math. Soc. vol. 41 (1949) pp. 87-91) have generalized Weber's parabolic cylinder function $D_n(x)$ by defining two functions as $\bar{D}_{km}(y) = \exp(y^k/2k)\, D^{(km)}_{(k)}\{\exp - (y^k/k)\}$ and $\bar{D}_{km+1}(y) = -\exp(y^k/2k) D^{(km+k-1)}_{(k)}\{\exp - (y^k/k)\}$, where $D^{(k)}_{(k)}$ stands for the operator $(d/dy)(1/y^{k-2})(d/dy)$ and $D^{(km)}_{(k)}$ means repetition of $D^{(k)}_{(k)}$ m times. Also $D^{(km+1)}_{(k)} = (d/dy)D^{(km)}_{(k)}$ and $D^{(km+k-1)}_{(k)} = (1/y^{k-2})(d/dy)D^{(km)}_{(k)}$. The functions $\bar{D}_{km}(y)$ and $\bar{D}_{km+1}(y)$ satisfy the recurrence relations $\bar{D}_{km}(y) - y^{k-1}\bar{D}_{km-k+1}(y) + (km - k + 1)\bar{D}_{km-k}(y) = 0$, $\bar{D}_{km+1}(y) - y\bar{D}_{km}(y) + km\,\bar{D}_{km-k+1}(y) = 0$. Starting with these we generate a class of polynomials analogous to Lommel polynomials but this class satisfies a recurrence relation of the kth order instead of the 2nd order. We also study a few properties of such a class. (Received September 20, 1957.)

539-10. L. C. Young: Approximate radius and approximate diameter.

Let $\mu$ be a measure on a set E in a metric space and for $0 < \epsilon \leq \mu(E)/2$,

let $r_\epsilon = \text{Inf } r(S)$ where $r(S)$ is the radius of a sphere S such that $\mu(E-S) < \epsilon$ and let $d_\epsilon = \text{Sup } d(X,Y)$ where $d(X,Y)$ is the distance of two subsets X, Y of E such that $\mu(X) \geq \epsilon$ and $\mu(Y) \geq \epsilon$. In certain spaces we define a function $N(y) > 0$ for $0 < y < 1$, such that $(1 - y)r_\epsilon \leq d_\epsilon$ wherever $\mu(E) \geq [N(y) + 1]\epsilon$. A particular case provides the crude but extremely useful little lemma: In Euclidean n-space we have $r_\epsilon \quad 2n^2 d_\epsilon$ wherever $\mu(E) \geq (n + 2)\epsilon$. (Received September 23, 1957.)

539-11. Withdrawn.

539-12. Shmuel Agmon: On coercive Dirichlet forms.

Let G be a bounded domain in $E^n$ and let $Q(u,u) = \int_G \sum_{|\alpha|, |\beta| \leq m} a_{\alpha\beta}(x) D^\alpha u \cdot \overline{D^\beta u} dx$ be an integral Dirichlet form of order m associated with an elliptic operator L of order $2m$. Let $\{B_j\}$ $1 \leq j \leq k$ be a system of k normal differential boundary operators ($0 \leq k \leq m$) of distinct orders between 0 and $m-1$. Denote by V the subset of functions of class $C_m$ in G which satisfy $B_j[u] = 0$, $1 \leq j \leq k$, on boundary G. Finally, let $Q_0(u,u)$ be a second Dirichlet form of order m which is coercive over V (f.i. $Q_0 = \int_G \sum_1^n |\partial^m u / \partial x_1^m|^2 dx$). Theorem: Q is coercive over V if and only if the (formally) self-adjoint boundary value problem associated with the Dirichlet form $\lambda Q + \mu Q_0$ and boundary operators $\{B_j\}$ for every $\lambda, \mu \geq 0$, $\lambda + \mu = 1$ is regular. Here regularity of an elliptic boundary value problem (not necessarily self-adjoint) is defined algebraically in terms of the differential operators, and is expressed by the nonvanishing of a certain determinant which one associates with every boundary point. The theorem contains results of Garding, Aronszajn and Schechter as special cases. It also characterizes all regular self-adjoint elliptic differential boundary value problems which are semi-bounded. (Received September 23, 1957.)

539-13. Edward Halpern: On the structure of hyperalgebras.

An A-hyperalgebra (A a commutative ring with unit e) consists of an A-module H, a product $\phi$: $H \otimes H \to H$, and a coproduct $\eta$: $H \to H \otimes H$ such that $\eta\phi = (\phi \otimes \phi)\rho(\eta \otimes \eta)$ where $\rho$ is defined by $\rho(x_1 \otimes x_2 \otimes x_3 \otimes x_4) = x_1 \otimes x_3 \otimes x_2 \otimes x_4$. It is associative if product and coproduct are associative; a unit $1 \in H$ is a unit for $\phi$ such that $\eta(1) = 1 \otimes 1$. An augmentation consists of an algebra homomorphism $\alpha : H \to A$ and a right inverse $\beta : A \to H$ such that $(i \otimes \beta\alpha)\eta(x) = x \otimes \beta(e)$, $(\beta\alpha \otimes i)\eta(x) = \beta(e) \otimes x$ ($i$ = identity on H). An element x in the kernel of $\alpha$ is

primitive if $\eta(x) = x\otimes 1 + 1\otimes x$; H is primitive if generated (under $\phi$) by primitive elements. For a graded A-hyperalgebra we require: H is graded, $H = \sum_{j\geq 0} H^j$; $\phi$ and $\eta$ are homogeneous of degree zero and $\rho$ is suitably signed; a standard augmentation, i.e., with $\varepsilon(A) = H^0$. Let $\theta(x\otimes y) = (-1)^{ij}\, y\otimes x$ (i,j degrees of x,y); then H is anticommutative if $\phi\theta = \phi$ (anticommutative product) and $\theta\eta = \eta$ (anticommutative coproduct). The following generalizes a theorem of H. Samelson: If H is a graded anticommutative associative K-hyperalgebra with unit over a field K of characteristic zero then it is primitive. (Received September 25, 1957.)

539-14. Roy Takenaga: Representation of Banach algebras by algebras of matrix-valued functions.

An $S_n$-algebra is a Banach algebra A such that for every regular maximal ideal M the quotient algebra A/M is isomorphic to $S_n$, where $S_n$ denotes the ring of all square matrices of order n over the complex number field. Let H denote the space of all bounded homomorphisms of A onto $S_n$. If $f\in A$ and $h\in H$, then $\hat{f}(h) = h(f)$ defines an algebra $\hat{A}$ of matrix-valued functions on H. H has a structure similar to fiber bundles; the structure space serves as base space, and cross-sections may be definable. Theorem: If A is such that (1) its H has a cross-section, (2) there exists an element f in A such that the spectrum of $\hat{f}(h)$ at each point h on a cross-section consists of n distinct points, and (3) given any cross-section C of H, there is a g in A such that $\hat{g}_{1j}\neq 0$ along C, for $j=1,\ldots,n$ then $\hat{A}$ has a closed subalgebra isomorphic to $S_n$. This theorem makes possible generalizations to $S_n$-algebras of most of the well known functional representation theorems in commutative Banach algebras. Appropriate definitions of self-adjoint and regular $S_n$-algebras are given. (Received September 26, 1957.)

539-15. Robert Hermann: Obstruction theory in fibre spaces, I.

Let $f:E\to B$ be a fibre space with fibre F, $\pi_1(B) = \pi_0(B) = \pi_0(F) = 0$, with B a CW complex, $B_n$ its n-skeleton. Let $g: B_n\to E$ be a cross-section. An n + 1 cocycle, with coefficients in $\pi_n(F)$ is defined which, in case the fibre space is a fibre bundle, actually reduces to the usual obstruction cocycle to extending g over $B_{n+1}$. The vanishing of this cocycle is necessary for the extension of g, and also sufficient, for most fibre spaces met in topological problems. All the usual properties of the extension cocycle carry over. The first obstruction

cohomology class is determined by the transgression homomorphism of the fibre space, in the usual way. If the fibre is a sphere or a complex projective space, the formulas of Liao and Kundert for the second obstruction are proved. The proof involves a more or less new general method for computing the higher obstructions in any fibre space, utilizing the Postnikov system of the fibre. The method is also applied, in the case where the fibre is a sphere, in computing some of the higher, still unknown, obstructions reduced mod p, where p is a prime number. The reduced p-powers of Steenrod are used in this computation. (Received September 26, 1957.)

539-16. Robert Hermann : Loop spaces of homogeneous spaces.

Let LCG be compact, simply connected Lie groups, L a subgroup totally non-homologous to zero mod $Z_p$. p is a prime number such that the Pontrjagin algebras $H_*(G,Z_p)$ and $H_*(L,Z_p)$ are Grassman algebras with odd-dimensional primitive elements as generators. Then, $H_*(\Omega(G/L),Z_p)$ is a polynomial algebra with even dimensional generators. Precisely, the fibre in the fibre space $(\Omega(G),\Omega(G/L),\Omega(L))$ is totally non-homologous to zero. If T is a toral subgroup of G, $H(\Omega(G/T),Z)$ is additively isomorphic to $H(T,Z)\otimes H(\Omega(G),Z)$. Let $P_n(K)$ denote the quaternion projective space of real dimension 4n. $H(\Omega(P_n(K),Z)$ is additively isomorphic to $H(S_3,Z)\otimes H(\Omega(S_{4n+3}),Z)$. Corollary: $\pi_k(P_n(K))\approx\pi_{k-1}(S_3)\oplus\pi_k(S_{4n+3})$. (Received September 26, 1957.)

539-17. E. J. Taft: The Whitehead first lemma for alternative algebras.

Let A be a finite-dimensional separable alternative algebra over a field of characteristic different from two or three. Let M be an alternative A-bimodule. $B = A\oplus M$ is an alternative algebra under $(a_1, m_1)(a_2,m_2) = (a_1a_2, a_1m_2 + m_1a_2)$ Let $R_b, L_b$ denote right and left multiplication by $b\in B$. If f is a derivation of A into M, then $f = R_g + L_g + \sum_i D_{x_i,y_i}$, where g is in the nucleus of M, $x_i\in A$, $z_i\in M$, and $D_{x,z} = [R_x,R_z] + [R_x, L_z] + [L_x, L_z]$. If the characteristic is zero, then g may be taken as zero. This generalizes the result for characteristic zero, due to Schafer (Trans. AMS, vol. 72, 1952). However, the method of proof is different. Schafer applied the analogous result of Jacobson for Jordan algebras to the Jordan algebra and module formed from A and M. Here, only the alternative structure theory is used. The proof consists of a reduction to special cases, the most important being the Cayley algebra, and the Cayley bimodule for a full two-by-two matrix algebra. The theorem has the usual corol-

lary that every derivation of a separable alternative algebra (characteristic not two or three) is inner (i.e., in the Lie algebra generated by the right and left multiplications). Also, the explicit result is false for characteristic two or three. (Received September 27, 1957.)

539-18. Nathaniel Coburn: The method of characteristics for a perfect compressible fluid in general relativity and nonsteady Newtonian mechanics.

By studying the discontinuity manifolds of the equations of motion and continuity of a perfect compressible fluid in general relativity, the characteristic partial differential equation and the relation between a normal vector to a characteristic surface, the unit time-like vector along a world line, and a bicharacteristic are determined. The normal vectors are shown to be space-like; the bicharacteristics are shown to be time-like; the vector along a world line (the generalized velocity vector) is the axis of the normal and bicharacteristic cones. By considering two families of characteristic surfaces (three-dimensional surfaces) which span space-time and which intersect in two-dimensional surfaces, it is shown that the generalized velocity vector and the normal vectors can be determined in terms of the tangent vectors of the two families of bicharacteristics, associated with the above characteristic surfaces, and two vectors fields which span the intersections of these surfaces (the first set of characteristic relations). With the aid of these equations, the second set of characteristic relations are determined. Then the characteristic first order partial differential equations are found in general relativity and by the usual approximation scheme, these relations are determined for the nonsteady Newtonian case. (Received September 27, 1957.)

539-19. M. W. Hirsch: On immersions and regular homotopy of differentiable manifolds.

An obstruction theory for immersions (regular maps) and regular homotopy is developed. (See Bull. Amer. Math. Soc. Abstract 63-3-380 for definitions.) Let K be a differentiable triangulation of a compact oriented manifold $M^n$. Let $V_{k,n}$ be the Stiefel manifold of n-frames in Euclidean k-space $E^k$, $k > n$. Let U be a neighborhood of the q-1 skeleton $K^{q-1}$, and $f: U \to E^k$ an immersion. An obstruction c(f) in $C^q(M^n; \pi_{q-1}(V_{k,n}))$ is defined with the following property: $c(f) = 0$ iff there is a neighborhood U' of $K^q$ and an immersion $f': U' \to E^k$ such that $f'|U' \wedge U\ f|U' \wedge U$. c(f) is a cocycle whose cohomology class w(f) is invariant

when f is replaced by an immersion of $K^{q-1}$ which agrees with f in a neighborhood of $K^{q-2}$. Obstructions to extending regular homotopies are also defined. When $k = n + q - 1$, $w(f)$ is identified with the normal Whitney class $W^q$ of $M^n$. The following theorems are proved: (1) Every compact $M^3$ can be immersed in $E^4$. (2) Two immersions $f,g: M^{2n} \to E^{4n}$ are regularly homotopic iff the corresponding normal classes are equal. The proofs depend strongly on recent work of Smale (Abstract no. 539-29). (Received September 30, 1957.)

539-20. E. E. Kohlbecker: Asymptotic properties of partitions.

Let $n(u)$ and $P(u)$ be functions on the non-negative real numbers such that $\int_0^R n(u)u^{-1}du$, $\int_0^R n(u)u^{-1}\log u\, du$, and $\int_0^R P(u)du$ exist in the Lebesgue sense for every positive R, and let $\exp(s\int_0^\infty (1 - e^{-us})^{-1} n(u)e^{-us}\, du) = s\int_0^\infty P(u)e^{-us}\, du$. Let $L(u)$ denote a slowly oscillating function in the sense of Karamata. If $\omega$ denotes a given positive number, we define $s_u$ to be a positive number such that $u = \omega\Gamma(\omega + 1)\zeta(\omega+1)(1/s_u)^{\omega+1} L(1/s_u)$. It is shown that $s_u$ exists and is determined up to a factor which tends to 1 as u tends to $\infty$ and also that $s_u \sim u^{\omega/(\omega+1)} L^*(u)$, where $L^*(u)$ is a slowly oscillating function related to $L(u)$. It is further shown that if $n(u) \sim u^\omega L(u)$ as u tends to $\infty$ and $P(u)$ is nondecreasing, then $\log P(u) \sim (1 + 1/\omega)u^{\omega/(\omega+1)} L^*(u)$ as u tends to $\infty$, and if $\log P(u) \sim (1 + 1/\omega)u^{\omega/(\omega+1)} L^*(u)$ as u tends to $\infty$ and $n(u)$ is nondecreasing, then $n(u) \sim u^\omega L(u)$ as u tends to $\infty$. (Received September 30, 1957.)

539-21. Garrett Birkhoff and R. S. Varga: Implicit alternating direction methods.

The Peaceman-Rachford method (SIAM vol. 3 (1955)), and the Douglas-Rachford method (Trans. Amer. Math. Soc. vol. 82 (1956)), both implicit alternating direction methods, have been shown to solve discrete numerical approximations of boundary value problems, for $-\nabla^2 u = S$ in a rectangle, much more rapidly than the Young (Trans. Amer. Math. Soc. vol. 76 (1954)) - Frankel (MTAC 4 (1950)) successive overrelaxation scheme. It has been shown by Young that the special case of $-\nabla^2 u = S$ in a rectangle is typical for successive overrelaxation applies to general self-adjoint elliptic equations. The main result is that no similar conclusion is true for implicit alternating direction methods; specifically, the case $-\nabla^2 u = S$ in a rectangle is essentially the only case where the Peaceman-Rachford, and Douglas-Rachford methods rigorously apply. A few weak positive results are also included. (Received September 30, 1957.)

539-22. J. H. Barrett: Nonoscillation of second-order linear differential equations with non-negative coefficients.

Recently, Z. Nehari (Trans. Amer. Math. Soc. vol. 85, pp. 428-445) established a criterion for disconjugacy of $y'' + f(x)y = 0$ on $a < x < \infty$, where $f(x)$ is positive and continuous, in terms of the least eigenvalue $\lambda(b)$ of the system: $y'' + \lambda f(x)y = 0$, $y(a) = y'(b) = 0$ for $a<b<\infty$. In fact, $\lambda(b)>1$ for $a<b<\infty$ is necessary and sufficient for disconjugacy. This result and others of Nehari's (with appropriate alteration) are obviously true for the more general equation (1) $(p(x)y')' + f(x)y = 0$, provided that (i) $\int^{\infty} 1/p = \infty$. However, for (ii) $\int^{\infty} f = \infty$ and (necessarily) (iii) $\int^{\infty} 1/p < \infty$, the above inequality is no longer necessary for disconjugacy and, for this latter case, $\lambda(b)$ grows as the reciprocal of $\int_a^b f$. Relations between (1) and its reciprocal equation (2) $(y'/f(x))' + y/p(x) = 0$ are studied and analogs of Nehari's necessary conditions for $p = 1$ are given for (1) with coefficients satisfying (ii) and (iii). A refinement (to non-negative coefficients $f(x)$) of Nehari's results plus the above extensions are accomplished by means of a detailed study of the angle equation of the Prüfer polar coordinate transformation. (Received September 30, 1957.)

539-23. R. G. Stoneham: A study of 60,000 digits of "e". Preliminary report.

In 1949, John von Neumann suggested that the ENIAC be used to calculate $\pi$ and "e" to many decimal places with a view toward obtaining a statistical measure of the randomness in the distribution of the digits. The work reported in (MTAC, vol. 4, 1950) showed that the value of the cumulative distribution function for the $\chi^2$-test as applied to $\pi$ revealed no conspicuous variations; however, "e" showed a sudden decrease in this value in the neighborhood of 2,000 places. Using 60,000 digits of "e" from a calculation performed on the ILLIAC by D. J. Wheeler, it is found that this unusual behavior does not occur again and the value of $\chi^2$ becomes fairly stable with $4 < \chi^2 < 9$ for regions farther out in the expansion, indicating a random character. In addition, definite evidence is exhibited that "e" is simply normal in the sense of Borel. (Received September 30, 1957.)

539-24. L. N. Howard: Hydrodynamic stability of a jet in two dimensional viscous flow.

The Orr-Sommerfeld problem for a jet in an unlimited viscous flow

($w = \operatorname{sech}^2 y$) can be converted into an integral equation eigenvalue problem by writing the equation in the form $(D^2 - \alpha^2)(D^2 - \omega^2)\phi = i\alpha R[w(D^2 - \alpha^2) - w'']\phi$, ($\omega^2 = \alpha^2 - i\alpha Rc$), inverting the operator on the left by aid of the Green's function, and reducing the $\phi''$ term by partial integration. Using this formulation one can then give a simple proof that the neutral curve (Im c = 0) is bounded away from R = 0 in the $\alpha$-R plane. Furthermore the integral equation eigenvalue problem can be solved by a systematic perturbation procedure going in powers of $\alpha$. By this means one can show that as $R \to \infty$ along the lower branch of the neutral curve, $\alpha R \to 0$ and in fact $\alpha R^2 \to 0.954$. Numerical calculations based on this approach lead to a minimum critical Reynolds number of about 4, although convergence is not too good at the minimum point. The method is not useful for the upper branch. (Received September 30, 1957.)

539-25. Leon Henkin: Weighted sums of squares.

There is a function $\lambda(m,n)$ such that any polynomial p in m variables, of degree n, with coefficients in some real closed field F and taking only non-negative values in F, can be expressed in the form $\sum_{i=1}^{\lambda(m,n)} C_i q_i^2$ where: $C_i$ is a positive element of F, $q_i$ is a rational function, and each $C_i$ and coefficient of $q_i$ can be computed from the coefficients of p by at most $\lambda(m,n)$ uses of the field operations. This strengthening of a famous result of Artin's (and generalization of a result of Abraham Robinson), and a result expressing any totally positive element of an algebraic number field as a sum of squares, are corollaries of the following metatheorem. For each formula $R(a_1 \ldots a_k; x_1 \ldots x_m; y)$ of the elementary theory of ordered fields, we can infer from the axioms for real closed fields and the formula $(\forall y)[(\exists x_1 \ldots x_n) R(a_1 \ldots a_k; x_1 \ldots x_m; y) \to y \geq 0]$ a formula of the form $(\forall x_1 \ldots x_n y)[R(a_1 \ldots a_k; x_1 \ldots x_m; y) \to T]$, where $T(a_1 \ldots a_k; x_1 \ldots x_m; y)$ is a disjunction of formulas $y = \sum_{i=1}^{r} C_i q_i^2 \wedge C_1 > 0 \wedge \ldots \wedge C_r > 0 \wedge D \neq 0$, and where each $C_i$ is a formal quotient of terms involving only $a_1, \ldots, a_k$; $q_i$ is a formal quotient involving $a_1, \ldots, a_k$, $x_1, \ldots, x_m, y$; and D is the formal product of all terms appearing as a denominator of some $C_i$ or $q_i$. (Received September 30, 1957.)

539-26. J. C. Holladay: On games with multiple moves.

Let G be a termination game (in the sense of "Cartesian Products of Termination Games", by J. Holladay, to appear in Annals game series vol. III). For a safe position p, define f(p) inductively as the smallest ordinal number $\sigma$

such that $\sigma \neq f(p')$ for any safe position p' following p. Let H be a termination game except that there may be several types of moves assigned to it. Let $G_1,\ldots,G_n$ be n termination games and let a move on $H \times G_1 \times \ldots \times G_n$ be defined as follows: Choose a subset of a given collection of subsets of $G_i$ $(i = 1,\ldots,n)$. This subset determines what type of move must be made on H. Each $G_i$ of the subset has one or more moves made on it and each $G_i$ not of the subset has one or no moves made on it. The last one to move wins. For each $G_i$, substitute one-pile ordinal Nim and let a move be made on the Nim game if and only if it corresponds to the $G_i$ in the chosen subset. Then a position $h,p_1,\ldots,p_n$ is safe in the former game if and only if each $p_i$ is safe and $h,f(p_1),\ldots,f(p_n)$ is safe in the latter game. (Received September 30, 1957.)

539-27. E. G. Straus: Sierpiński sets in groups.

A Sierpiński set is a set congruent to each subset obtained by deleting a single element. If congruence is defined in terms of the group operation we prove that a group contains a Sierpiński set if and only if it contains a free subgroup of rank 2. (Received September 30, 1957.)

539-28. Ernest Michael: Cross sections in certain topological groups.

Theorem: If G is a metrizable topological group, and H a closed subgroup of G which is (locally) isomorphic to the additive group of a Banach space, then there exists a (local) cross section. This generalizes a result of R. Bartle and L. M. Graves (Trans. Amer. Math. Soc. vol. 72 (1952) pp. 400-413), where G is a Banach space, and H a linear subspace. (Received September 30, 1957.)

539-29. Stephen Smale: The classification of immersions of spheres in Euclidean spaces.

An immersion (See Smale, Abstract no. 380, Bull. Amer. Math. Soc. vol.63, no.3) $f:S^k \to E^n$, $n>k$, is a based immersion if $f_*(x_0) = y_0$ where $x_0,y_0$ are fixed k-frames of $S^k, E^n$ respectively. A based regular homotopy is a regular homotopy (Smale, ibid.) which at every stage is a based immersion. If f and g are based immersions, an invariant $\Omega(f,g) \in \pi_k(V_{n,k})$ is defined ($V_{n,k}$ is the Stiefel manifold of k-frames in $E^n$). Then: if f and g are $C^\infty$ based immersions of $S^k$ in $E^n$ they are based regularly homotopic if and only if $\Omega(f,g) = 0$. Let $\Omega_0 \in \pi_k(V_{n,k})$ and let a based $C^\infty$ immersion $f:S^k \to E^n$ be given. Then there exists a based immersion $g:S^k \to E^n$ such that $\Omega(f,g) = \Omega_0$. Thus there is a 1-1

correspondence between elements of $\pi_k(V_{n,k})$ and based regular homotopy classes of immersions of $S^k$ in $E^n$. As an application it is proved that $S^k$ may be immersed in $E^{k+1}$ with arbitrary normal degree (Milnor, Comm. Math. Helv. vol. 30, pp. 275-283) if and only if $S^k$ is parallelizable. This answers a question of Milnor (ibid.). Also, two immersions of $S^k$ in $E^{2k}$ are regularly homotopic if and only if they have the same intersection number. (Whitney, Ann. of Math. vol. 45, pp. 220-246). This partially answers a question of Whitney (ibid). (Received September 30, 1957.)

539-30. Alfred Tarski: Models of universal sentences in predicate logic with infinitely long formulas.

Various model-theoretical results for ordinary predicate logic $P_0$ extend to (first-order) predicate logics with infinitely long expressions. Let, e.g., $P_1$ be predicate logic with atomic formulas consisting of predicates followed by finitely many variables, and compound formulas obtained by (i) forming disjunctions and conjunctions of denumerable (= at most denumerable) sequences of formulas, (ii) preceding a formula by a quantifier with denumerably many subscripts or a negation symbol. The terms "universal sentence", "model" are defined as in $P_0$. Author's result, Indag. Math. 16, p. 578, generalizes as follows: A class K of relational systems is the class of models of some set of universal sentences in $P_1$ iff two conditions hold: if $\mathfrak{A} \in K$, then K contains all isomorphs and subsystems of $\mathfrak{A}$; if K contains all denumerable subsystems of $\mathfrak{A}$, then $\mathfrak{A} \in K$. Examples: (1) the class of all well ordered systems $\langle A, \prec \rangle$, characterized by axioms of simple ordering and one infinitely long axiom, $\forall_{x_0,\ldots,x_n,\ldots}[(x_0 \leqq x_1) \vee \ldots \vee (x_n \leqq x_{n+1}) \vee \ldots]$; (2) the class of all scattered ordered systems. Neither of these classes is characterizable in $P_0$. The result naturally extends to predicate logics $P_\alpha$ for arbitrary ordinals $\alpha$; in constructing $P_\alpha$, arbitrary sequences (of variables and formulas) with length $< \omega_\alpha$ are used. (Received September 30, 1957.)

539-31. L. W. Anderson and A. T. Bharucha-Reid: Topological semigroups and n-means.

Let S be a commutative topological semigroup and for each $n \geqq 2$ let $M_n$ be an n-mean on S such that $xM_n(x_1,x_2,\ldots,x_n) = M_n(xx_1,xx_2,\ldots,xx_n)$. For each x in S, let $F_n(x) = M_n(x,x^2,x^3,\ldots,x^n)$. We assume that if the net $F_n(x)$ converges to y then $xF_n(x)$ also converges to y. Result: If $F_n(x)$ converges to y then

$xy = y = y^2$. If e is an idempotent in S, let C(e) denote the set of all x in S such that $F_n(x)$ converges to e. Result: If $F_n(x)$ converges for all x in S then C(e) is an ideal if, and only if, e is a zero. (Received October 1, 1957.)

539-32. František Wolf: Extension of the spectral decomposition theorem.

We investigate operators T in Banach space which have their spectrum $\sigma(T)$ on the unit circle. Let $\mathscr{L}(T)$ be a Banach algebra of functions on $|\lambda| = 1$, such that there exists an operational calculus $f \to f(T)$. Suppose there exists a mapping $S \to \mathscr{M}(S)$ of a certain class C of subsets of $\sigma(T)$ into subspaces of the Banach space that are invariant under T. If T is unitary in a Hilbert space, then $\mathscr{L}(T)$ may be taken to be the ring of bounded B-measurable functions and C, the class of B-measurable sets. Then $f \to f(T)$ is local insofar that a change of f inside of an S affects f(T) only in $\mathscr{M}(S)$. This local behavior is the basic fact of the spectral decomposition theorem. If $R(\lambda, T) = O(|1 - |\lambda||^{-n+2})$, then it has been shown (Indag. Math. vol. 19 (1957) p. 302) that $\mathscr{L}(T)$ may be taken as $C^n$, the n times continuously differentiable functions. C is the class of closed sets. Under these definitions the local behavior of the mapping is preserved. Further, if $\int_{1/2}^{2} \log^+\log^+ \max_{\phi} \|R(re^{i\phi}, T)\| \, dr < \infty$, then the situation remains unchanged. $\mathscr{L}(T)$ contains "local functions" and C contains all intervals. In one case, when the last integral diverges, it can be proved that $\mathscr{L}(T)$ is a quasianalytic class and the local behavior breaks down. (Received October 1, 1957.)

539-33. D. H. Lehmer: Compositions of basic divisor functions.

Let $\sigma_k(n)$ denote the sum of the k-th powers of all the divisors of n. The functions $n^{\alpha}\sigma_{\beta}(n)$, where $\alpha$ and $\beta$ are non-negative integers and $\beta$ is odd, are called basic divisor functions. Ramanujan noted the formula $12 \sum \sigma_1(k)\sigma_1(n-k) = 5\sigma_3(n) - (6n - 1)\sigma_1(n)$, the sum being for $k = 1(1)n - 1$. This is the simplest of 7 such additive-multiplicative theorems giving the composition of two $\sigma$-functions as a linear combination of basic divisor functions. In this paper we consider the composition of any number of basic divisor functions and give all 35 of the cases in which the result is a linear combination of such functions. These formulas, including the 7 of Ramanujan, may be grouped according to a natural system of weighing. There are then 1,3,9,19,3 formulas of weights 4,6,8,10,14 respectively. It is essential that $\beta$ be odd. There is no theory when $\beta$ is even. (Received October 1, 1957.)

539-34. Edgar Reich: Solution of the Takács integrodifferential equation.

While studying a Markov process of mixed type, Takács (Acta Math. Hung. vol. 6 (1955) pp. 101-129) encountered the integrodifferential equation (*) $F_t = F_x - r(t)F + r(t)\int_0^x H(x-y)d_y F(t,y)$, $x,t \geq 0$, where $F=F(t,x)$ is a distribution function in $x \geq 0$, for $t \geq 0$, and H is a distribution function, $x \geq 0$, and $r(t) \geq 0$ is continuous. It is known that (*) can be solved by quadratures, if $P(t)=F(t,0)$ is known. If $H(x) = \int_0^x h(u)du$, $e^{-cx}h(x) \in L^2(0,\infty)$ for some $c \geq 0$, then we can show that P(t) is the unique continuous solution of a Volterra equation of the second kind with kernel $d/dt\ P.V.(2\pi i)^{-1}\int_{x-i\infty}^{x+i\infty} \exp\{(t-u)s-[1-\Psi(s)]\cdot[R(t)-R(u)]\}ds/s$, $x>c$, $R'(t)=r(t)$, $\Psi(s) = \int_0^\infty e^{-sx}h(x)dx$. A similar integrodifferential equation of Beneš related to the return-to-zero times of the Markov process in question is solved similarly. (Received October 1, 1957.)

539-35. H. E. Salzer and G. M. Kimbro: Tables for bivariate osculatory interpolation over a Cartesian grid.

Formulas are developed for binary polynomials P(x,y) which agree together with the partial derivatives $P_x(x,y)$ and $P_y(x,y)$, with $f \equiv f(x,y)$, $f_x \equiv f_x(x,y)$ and $f_y \equiv f_y(x,y)$ at n specified points. They have the advantage over ordinary bivariate interpolation of packing 3n conditions into n points. Unlike univariate polynomial osculatory interpolation which always possesses a solution for any irregular configuration of fixed points, a binary polynomial of prescribed form may not satisfy those 3n conditions for any choice of interpolation points, or may fail for just certain special configurations. Explicit formulas or methods are developed for the general 2- to 5-point cases. For interpolation over any square Cartesian grid of length h, for suitable 2- to 5-point configurations of $(x_i,y_i)$, according to the formula (1) $f(x,y) \equiv f(x_0+ph, y_0+qh) \sim P(x_0+ph, y_0+qh) = \sum_{i=0}^{n-1}\{A_i^{(n)}(p,q)f_i + h[B_i^{(n)}(p,q)f_{x_i} + C_i^{(n)}(p,q)f_{y_i}]\}$ tables of exact values of $A_i(p,q)$, $B_i(p,q)$ and $C_i(p,q)$ are given for p and q each ranging from 0 to 1 at intervals of 0.1. A closed expression for the remainder in (1) has not been found. In its place, formulas are derived for the leading terms in the bivariate Taylor expansions for the remainders. These formulas should cut down the number of needed strips in the numerical solution of Cauchy's problem for first order partial differential equations by the method of characteristic strips. (Received October 3, 1957.)

539-36. Rufus Isaacs: Search games with immobile hiders.

The Evader distributes h hiders among the N points of a connected linear graph. The Pursuer places s searchers on points and on subsequent moves each is displaced to an adjoining point. A hider is captured when coincident with a searcher. The Pursuer knows where a hider is only when it is captured. The payoff is the number of his moves when all hiders are captured. It is proved that $(h/(h+1))(N/s) < \text{value} \leqq N/s$. Thus when h is large, any search strategy is nearly optimal if no points are duplicated. When $h = 1$, solutions are easy. For reasonable graphs, the solution is known when $s = 1$ as well as approximately optimal strategies for arbitrary h and s. (Received October 2, 1957.)

539-37. R. W. Preisendorfer: Principles of invariance and the planetary hydrosphere problem.

The planetary hydrosphere problem in radiative transfer theory has as its physical counterpart the problem of the penetration of natural light into natural hydrosols (oceans, lakes, rivers, etc.). The corresponding mathematical problem runs as follows: given a slab $X = \{x: 0 \leqq z \leqq z_1\}$ (i.e., a subset of euclidean three space $E_3$ between two parallel planes) of optical depth $\tau_1$, with reflecting (upper and lower) boundaries, and with attenuation and scattering functions $\alpha$, $\sigma$ defined in the slab. If an incident radiance distribution $N^0$ is defined on the upper boundary, required: the radiance distributions N at all optical depths $\tau$, $0 \leqq \tau \leqq \tau_1$. It is shown that, by means of the principles of invariance (as originated by Ambarzumian and developed by Chandrasekhar) that the complete two boundary problem is formulable as a Fredholm integral equation of the second order, second kind: $N(\tau, \underline{\xi}) = N_0(\tau, \underline{\xi}) + (1/4\pi)\underline{\xi}\cdot\underline{k}) \cdot \int_{\Omega_+} K(\tau; \underline{\xi}; \underline{\xi}')N(\tau, \underline{\xi}')\, d\Omega(\underline{\xi}')$, where $K(\tau; \underline{\xi}; \underline{\xi}'') = (1/4\pi) \int_{\Omega_+} S(\tau_1 - \tau; \underline{\xi}, \underline{\xi}') \cdot S(\tau; \underline{\xi}'; \underline{\xi}'')(1/\underline{\xi}'\cdot\underline{k})d\Omega(\underline{\xi}')$, and the S-function is the diffuse reflectance function of general radiative transfer theory. This formulation is particularly adaptable to computations by large scale automatic computors. (Received October 2, 1957.)

539-38. F. W. Anderson: Concerning a generalization of the Stone-Weierstrass approximation theorem. Preliminary report.

Let X be a completely regular space and C(X) the algebra of all real-valued continuous functions on X. Let $A \subseteq C(X)$; then A is said to be: (i) pseudo-normal in case whenever $F_1$ and $F_2$ are completely separated subsets of X there is a bounded function $f \in A$ such that $f \leqq 0$ on $F_1$ and $f \geqq 1$ on $F_2$; (ii) normal in

case $\{f \in A;\ 0 \leq f \leq 1\}$ is pseudo-normal; (iii) LF-complete in case for every subset $\{f_\alpha\}$ of A such that the family $\{X - Z(f_\alpha)\}$ is locally-finite in X, the continuous function $\sum f_\alpha$ is in A. If $A \subseteq C(X)$, then L(A) denotes the LF-completion of A in C(X), and $A^u$ ($A^m$) denotes the uniform closure (m-closure) of A in C(X). Typical results: If A is a pseudo-normal subalgebra of C(X), then $L(A^u) = C(X)$. If A is a normal subalgebra of C(X), then $L(A^u) = (L(A))^u = (L(A))^m = C(X)$. Several related results, including partial converses of the above results, are also obtained. (Received October 2, 1957.)

539-39. F. W. Anderson and R. L. Blair: Characterizations of the algebra of all real-valued continuous functions on a completely regular space.

If X is a topological space, then C(X) denotes the set of all real-valued continuous functions on X. For X compact, a variety of characterizations of C(X) is known. For X an arbitrary completely regular space, however, no such characterization of C(X) has previously been given. In this paper several characterizations of C(X), for X completely regular, are obtained. In particular, C(X) is characterized as an algebra, as a lattice-ordered algebra, and as a lattice-ordered vector space. In addition, new characterizations of C(X) are given in the case where X is compact. Other results include characterizations of certain important subalgebras of C(X). (Received October 2, 1957.)

539-40. C. C. Elgot and J. B. Wright: Series-parallel graphs and lattices.

A path in a doubly rooted graph $\langle G, p_1, p_2 \rangle$ is a sequence of distinct points in G, initiating with $p_1$ and terminating with $p_2$. The link relation of $\langle G, p_1, p_2 \rangle$ is the set of ordered pairs of points which are consecutive in some path. A graph is admissible if each of its points is in some path and for each edge of the graph with end points a, b, either $\langle a,b \rangle$ or $\langle b,a \rangle$ is an element of its link relation. Theorem: If $\langle G, p_1, p_2 \rangle$ is admissible, then it is series-parallel if and only if its link relation is asymmetric. This theorem is proved by obtaining intermediate results which characterize the transitive closure of the link relations of series-parallel graphs (as series-parallel lattices). One such characterization is: A lattice is series-parallel if and only if: given three elements such that (at least) two of the three pairs of elements are incomparable then (at least) two of the three joins (meets) of pairs are equal. (Received October 2, 1957.)

539-41. I. S. Gál: On the completion of uniform structures.

The object is to give new methods of completion of uniform structures which are more suitable for functional analytic purposes than known methods. Let $C(S,\mathcal{U})$ be the ring of bounded real valued functions on the set S which are uniformly continuous relative to the uniform structure $\mathcal{U}$ for S. Denote by X the set of all real maximal ideals in $C(S,\mathcal{U})$. A uniform structure $\mathcal{V}$ can be introduced on the set X by specifying a family of pseudo-metric structures $\mathcal{V}_d$ and setting $\mathcal{V} = \text{lub}\{\mathcal{V}_d\}$: Define $\mathcal{V}_d$ by extending the bounded pseudo-metric d of the gage of $\mathcal{U}$ as follows: $d(I_1,I_2) = \text{lub}\,[|I_1(d_t) - I_2(d_t)| : t \in S]$ where $I_i(d_t)$ is the image of $d_t$ in $C(S,\mathcal{U})/I_i$ and $d_t$ is given by $d_t(s) = d(t,s)$. Theorem: The restriction of $\mathcal{V}$ to the closure of S in X relative to the uniform topology generated on X by $\mathcal{V}$ is the completion of $\mathcal{U}$. The completeness of this structure can be proved by using the following Lemma: If $\mathcal{F}$ is a Cauchy filter in S relative to $\mathcal{U}$ then for every $F_0 \in \mathcal{F}$ and $U_0 \in \mathcal{U}$ there is a non-negative bounded real valued function x on S such that x is uniformly continuous relative to $\mathcal{U}$; $x(\mathcal{F})$ is convergent to 0; and $x(s) = 1$ for every $s \notin U_0[F_0]$. The Stone-Čech compactification appears as a special case of completion. (Received October 2, 1957.)

539-42. Olga Taussky: On a matrix theorem of Craig and Hotelling.

The theorem in question is: Let A and B be two $n \times n$ symmetric matrices such that $\det(I - \lambda A)\times\det(I - \mu B) = \det(I - \lambda A - \mu B)$ for all $\lambda$ and $\mu$. Then $AB = 0$. This is now reproved and generalized by noting that A and B have the L-property for pairs of matrices. The theorem does not hold for arbitrary matrices. (Received October 2, 1957.)

539-43. John Todd: The condition of certain matrices. III.

The P-condition number of matrices associated with certain discretizations of the two-dimensional Laplacian operator are determined. They give some indication of the errors to be expected in the numerical solution of the discretized Dirichlet problem. Although the local truncation error for a certain nine-point discretization is $O(h^4)$ as compared with $O(h^2)$ for the usual five-point discretization, the associated condition numbers are of the same order, approximately $(16/3)\pi^{-2}n^2$ and $4\pi^{-2}n^2$. (Here h is the mesh length and $nh = 1$.) The results of J. Williamson (Bull. Amer. Math. Soc. (1931) vol. 37 pp. 585-590) concerning the characteristic values of partitioned matrices, in which all the blocks -ational functions of a single matrix, are used extensively. (Received Octob 1957.)

539-44. Alexander Weinstein: On convex functions.

Let $r \geq 0$. If $v(r)$ is convex in $r$, then $r^{-1}v(r)$ is convex in $r^{-1}$. If $v(r)$ is non decreasing and convex in $r$, then $r^c v$ is convex in $r^c$ for $c > -1$, $c \neq 0$. If $v(r)$ is non decreasing and $r^b v$ is convex in $r^b$, then $r^a v$ is convex in $r^a$ for $a > b$, $a \neq 0$. As an application let us consider the mean value M of a subharmonic function $f(x_1, x_2, \ldots, x_m)$, $m \geq 3$, over a solid sphere of radius r. It is known that M is convex in $r^{2-m}$ and non decreasing for r increasing. Therefore $r^{m-2}M$ is convex in $r^{m-2}$. From this follows that $r^m M$ is convex in $r^m$, a result previously obtained by the author (A. Weinstein, Annali di Matematica (1957) vol. 43, pp. 325-340). (Received October 2, 1957.)

539-45. C. B. Morrey, Jr: The analytic embedding of abstract real-analytic manifolds.

In this paper, it is proved that any abstract compact real-analytic $\nu$-dimensional manifold $M_0$ can be embedded analytically in a Euclidean space of sufficiently high dimensionality. This is done as follows: $M_0$ is embedded in a complex analytic extension M on which a $C^\infty$ Hermitean metric is introduced which is real on $M_0$. For R positive, $M_R$ denotes all points of M whose geodesic distances from $M_0 \leq R$; if $R \leq R_0$, $\partial M_R$ is $C^\infty$. $T_R w = \Delta w$ $(= d\bar{\partial} w)$ where $D(T_R)$ is all $w \in C^\infty$ in $M_R$ with $\bar{\partial} w$, $\Delta w$ $C^\infty$ on $\overline{M}_R$ and $n\bar{\partial} w = 0$ on $\partial M_R$. $A_R$ is all analytic w in $L_2$ on $M_R$. It is shown that if $R \leq R_1$, $w \in D(T_R)$, $w_1$ is its projection on $L_2 \ominus A_R$, then $w - w_1 \in A_R$, $w_1 \in D(T_R)$, $\|w_1\| \leq CR^2 \|\Delta w\|$. If $P_0$ is any point on $M_0$, an analytic vector $\{w_{2\alpha}\}$ with nonzero linearly independent gradients at $P_0$ is formed by constructing certain approximately analytic vectors $(w_\alpha)(\|\Delta w_{1\alpha}\| < \epsilon(R))$, analytic near $P_0$ with $\partial w_{1\alpha}/\partial z^\beta = \delta_{\alpha\beta}$ there and subtracting off their projections $w_{1\alpha}$. (Received October 2, 1957.)

539-46. S. W. Golomb: A connected topology for the integers.

A topology D for the positive integers is obtained when those arithmetic progressions $\{an + b\}$ with $(a,b) = 1$ are taken as a basis for the open sets. In this topology, which is connected and Hausdorff, the infinitude of the primes is easily demonstrated. Moreover, the statement that the primes are a dense subset of the integers is equivalent to Dirichlet's famous theorem concerning primes in arithmetic progressions. (Received October 2, 1957.)

539-47. K. L. Cooke: The rate of increase of real continuous solutions of certain algebraic functional equations.

Let $P(t,u,v)$ denote a polynomial in the variables $t,u,v$, with real coefficients, and let $\delta(t)$ be a positive, continuous function for which $t + \delta(t)$ is monotone increasing for $t \geq a$. It is shown that every real solution $u(t)$ of $P(t,u(t), u(t + \delta(t)))=0$, continuous for all $t \geq b \geq a$, must satisfy $\liminf_{t \to +\infty} |u(t)|/f(t) = 0$; and further, that if $u(t)$ is also monotone, then $\limsup_{t \to +\infty} |u(t)|/f(t) = 0$. Here $f(t)$ is a certain positive, continuous, non-decreasing function which depends on $P$ and $\delta$, but not on the particular solution $u$ in question. The "best possible" form of $f(t)$ is found for the three special cases $\delta(t)=1$, $\delta(t)=(q-1)t$ $(q>1)$, and $\delta(t)= t^q - t$ $(q > 1)$. (Received October 3, 1957.)

539-48. Franklin Haimo: Stability groups as splitting extensions, Preliminary Report.

Let $A_i$ be abelian groups, and let $S_0 = (1)$, $S_j = A_1 \oplus A_2 \oplus \cdots \oplus A_j$. The stability group $T_n$ of the chain $S_0 \subset S_1 \subset S_2 \subset \cdots \subset S_n$ is that group of automorphisms of $S_n$, each member of which induces the identity on the quotient $S_j / S_{j-1}$, $j = 1,2,\ldots,n$. One can show that $T_n$ is a splitting extension of the group of 1-cocycles of $A_n$ with coefficients in $S_{n-1}$ by $T_{n-1}$. Suppose that $G$ is a splitting extension of $S_n$ by a group $B$ where the induced $B$-operators on $S_n$ are in the centralizer of $T_n$. Then the stability group of $G \supset S_n \supset \cdots \supset S_0$ is a splitting extension of $G \supset S_n \supset S_0$ by $T_n$. (Received October 3, 1957.)

539-49. Dr. W. T. Kyner: On the computation of a cut.

In a study of the kinetics of polymerization, it is required to solve an infinite system of ordinary differential equation of the form $dX_n/dt = \gamma X_{n-1} - X_n$, $n \geq 1$, $\gamma$ constant, with initial conditions, $X_n(0) = 0$. $X_0(t)$ is a solution to a system of finite order which will not be discussed here. The object of interest is a cut, i.e., $X_n(t)$ for fixed $t$ and varying $n$. Since $X_0$ is a finite sum of exponentials with polynomial coefficients, $X_n$ can be written in similar form with the exponentials replaced by incomplete gamma functions. A recurrence relation can, therefore, be obtained which permits the computation of a cut without knowledge of the solutions for earlier values of $t$. (Received October 4, 1957.)

539-50. J. R. M. Radok: The partial differential equation analogue of an infinite system of ordinary differential equations.

The use of finite difference equations replacing partial differential equations for the purpose of numerical computations is well known. The reverse process for dealing with infinite systems of ordinary equations has, to the author's knowledge, less often been applied. An example of a system of equations is presented which occurs in the kinetics of polymerization and the solution of the partial differential equation analogue of which exhibits the same qualitative features as that of the infinite system, while being analytically simpler. (Received October 4, 1957.)

539-51. O. A. Kreyszig: On partial differential equations of the fourth order.

Let the coefficients of (1) $\Delta\Delta w + A(x,y)w_{xx} + B(x,y)w_{xy} + C(x,y)w_{yy} + D(x,y)w_x + E(x,y)w_y + H(x,y)w \equiv u_{zzz^*z^*} + a(z,z^*)u_{zz} + b(z,z^*)u_{zz^*} + c(z,z^*)u_{z^*z^*} + d(z,z^*)u_z + e(z,z^*)u_{z^*} + h(z,z^*)u=0$, $z=x+iy$, $z^*=x-iy$, be entire functions of the respective complex variables. Then the solutions of (1) can be represented by means of Bergman operators, cf. Bergman, Duke Math. J. vol. 11 (1944) p. 617. In this way properties of $u(z,z^*)$ can be obtained from the coefficients $u_{m0}, u_{m1}, m=0,1,\ldots$, of the development (2) $u(z,z^*) = \sum_{m,n=0}^{\infty} u_{mn} z^m z^{*n}$. Obviously, similar possibilities should exist if other subsequences $u_{mn}, n > 1$ fixed, $m=0,1\ldots$, are known. The problem thus arises to derive relations between these coefficients and the above ones. By representing (2) in the form (2') $u(z,z^*) = \sum_{n=0}^{\infty} u_n(z) z^{*n}$, $u_n(z) = \sum_{m=0}^{\infty} u_{mn} z^m$, and inserting (2') into (1) a system S of ordinary differential equations of the second order is obtained, relating the above subsequences of coefficients. At singular points of $u_n$, $n > 1$, at least one of the functions $u_0, u_1$ is singular. The converse may not be true, i.e. the intersection of the domain of regularity of $u_0$ and $u_1$ may be a proper subdomain of the domain of regularity of $u_n$. By eliminating $u_2, \ldots, u_{n-1}$ from S the domain of regularity and other properties of $u(z,z^*)$ can be obtained in terms of the coefficients $u_{mn}, n > 1$ and fixed, $m = 0,1\ldots$ . (Received October 9, 1957.)

539-52. Emma Lehmer: Conditions for quartic residuacity.

Let $\sigma_k$ be the sum of k-th powers of the roots of the reduced quartic period equation, then a condition for quartic residuacity can be stated as follows: If a prime q is a quadratic residue of $p = 4n + 1$, then it is also a

quartic residue of p if and only if $\sigma_{q+1} \equiv 12p \pmod q$ in case $p = 8n+1$ and $\sigma_{q+1} \equiv -4p \pmod q$ in case $p = 8n+5$. Considering the period equation as a congruence modulo p we can actually calculate $\sigma_{q+1} \pmod q$ and obtain the following criteria. Let $p = a^2 + b^2$, a odd, then if $a \equiv 0 \pmod q$, q is a quartic residue of $p = 8n+1$ if and only if 2 is a quadratic residue of q, and of $p = 8n+5$ if and only if -2 is a quadratic residue of q. If $a \not\equiv 0 \pmod q$, there exists a $\lambda$ such that $p \equiv a^2 \lambda^2 \pmod q$. Then q is a quartic residue of $p = 8n+1$ if and only if both $2\lambda(\lambda+1)$ and $2\lambda(\lambda-1)$ are quadratic residues of q and q is a quartic residue of $p = 8n+5$ if and only if both $-2\lambda(\lambda+1)$ and $-2\lambda(\lambda-1)$ are quadratic residues of q. In particular if q divides b, then $\lambda = 1$ and q is a quartic residue of all primes of the form $p = a^2 + q^2\beta^2 = 8n+1$, but q is a quartic residue of primes p of the form $p = a^2 + q^2\beta^2 = 8n+5$ if and only if $q = 4n+1$.

(Received November 15, 1957.)

## THE NOVEMBER MEETING IN CORAL GABLES, FLORIDA

November 29-30, 1957

540-1. D. W. Wall: Socles and UMFR algebras.

Let $\mathcal{A}$ be a finite dimensional algebra over a field. If $\mathcal{A}$ has only one faithful representation minimal with respect to being faithful then $\mathcal{A}$ is a UMFR algebra. If $\mathcal{Y}$ is an ideal (left or right) of $\mathcal{A}$ then the socle $\mathcal{S}(\mathcal{Y})$ of $\mathcal{Y}$ is the sum of all minimal subideals of $\mathcal{Y}$. Let $\mathcal{A}$ be a UMFR algebra and $\mathcal{Y}$ any primitive ideal of $\mathcal{A}$. The socle $\mathcal{S}(\mathcal{Y})$ can be written as a direct sum $\sum_{i=1}^{r}\sum_{j=1}^{g(i)} \mathcal{M}_{ij}$, $\mathcal{M}_{ij} \cong \mathcal{M}_{hk}$ if and only if $i = h$. Since $\mathcal{A}$ is UMFR, each $\mathcal{M}_{ij}$ is $\mathcal{A}$-isomorphic to the unique minimal subideal of some dominant ideal $\mathcal{Y}_{ij}$. Let the $\mathcal{A}$-module $\mathcal{N} = \sum_{i=1}^{r} \sum_{j=1}^{g(i)} \mathcal{Y}_{ij}$ be the direct sum of the $\mathcal{Y}_{ij}$. Let $\mathcal{A} = \sum_{i=1}^{n} \sum_{j=1}^{f(i)} \mathcal{P}_{ij}$, $\mathcal{P}_{ij} \cong \mathcal{P}_{hk}$ if and only if $i = h$, be a decomposition of $\mathcal{A}$ written as a direct sum of primitive ideals chosen so that $\mathcal{P}_{ij} \cong \mathcal{P}_{hk}$ if and only if $i = h$. Consider the following conditions upon the socles of all primitive ideals $\mathcal{Y}$ of $\mathcal{A}$: (1.) $g(i) \leqq f(i)$; (2.) $r \neq 1$; (3.) $r = 1$ and $g(1) \leqq f(1)$; (4.) $g(i) = 1$; (5.) $r = 1$ and $g(1) = 1$. Each of these conditions determines a subclass of the UMFR algebras. These subclasses are among those previously defined (Bull. Amer. Math. Soc. Abstract 63-4-418). (Received September 20, 1957.)

540-2. M. A. Dengler: Derivative coefficients for the numerical solution of partial differential equations.

Finite difference approximations for partial derivatives are established by use of appropriate derivative coefficients which are intended to facilitate the numerical solution of boundary value problems involving elliptic or hyperbolic differential equations. The terms derived are associated with simple point patterns, are general in nature, and apply to nonlinear equations as well. Among the numerical expressions studied are one-dimensional derivative coefficients for first, second, third and fourth derivatives in the vicinity of walls (boundaries), listed to an accuracy of nineteen significant figures. Specific expressions are established for the finitie difference analysis of equations in the neighborhood of curved boundaries as are most frequently encountered in technical physics. (Received September 27, 1957.)

540-3. R. J. Koch: Note on weak cutpoints in clans.

Let S be a clan, i. e., S is a continuum which is a topological semigroup with unit u, and suppose S is not a group. Let $T_u$ be the set of points at which S is not aposyndetic with respect to u (i.e., $T_u$ is the set of all x such that if M is a continuum with $x \in M^0$, then $u \in M$). It is conjectural that $T_u = u$, and this is shown to hold in case S is either one-dimensional or homogeneous. Also $T_u \neq u$ if and only if u is a weak cutpoint. Further, if $T_u \neq u$, then $T_u$ is a c-set which coincides with the identity component of the maximal subgroup containing u. Finally, if S is one-dimensional, then u does not cut any subcontinuum of S. (Received September 30, 1957.)

540-4. Diran Sarafyan: An infinite set or class of formulas for the computation of roots.

After defining finite and infinite uniform continued fractions and showing their geometric significance, the following symbolic representation is introduced for them $C_0(k/a) = 0$; $C_1(k/a) = k/a$; $C_2(k/a) = k/(a + k/a)$; etc. It is shown that $r_i = a + 2C_i(k/a)$ where $r_i$ designates the i-th approximation to $r = n^{1/2}$ and $k = (n - a^2)/4$. An error analysis indicates that if $r_0$, the starting approximation, is correct to d-digits then $r_j$ is correct to (j + 1)d-digits. Combining $r_1 = a + 2k/a$ and $k = (n-a^2)/4$ the so called Newton's formula is obtained. Thereafter the author establishes the general formula $r_1 = (\alpha n/r_0 + \beta r_0)/(\alpha + \beta)$ defining a class or an infinite set of formulas since $\alpha$ and $\beta$ are any two rational numbers but such that $\alpha \neq \beta$. It is readily seen that Newton's formula is a member of this class (case $\alpha = \beta$). However from the computational as well as fast convergence stand point by no means is it the best member of this class. Finally these results are further generalized by considering the case $n^{1/m}$, $m = 2,3,\ldots$ and establishing the most general formulas $r_1 = (1/(\alpha + \beta))$ $\cdot(\alpha n/r_0^{m-1} + \beta r_0)$, $(m - 1)\alpha \neq \beta$ which define a class of formulas. Letting $\alpha = r_0^{m-1}$ and choosing $\beta$ in a manner that $(\alpha + \beta)$ becomes an integral power of ten or a convenient multiple of it, the operation of division is completely eliminated from the computations. (Received September 30, 1957.)

540-5. Jack Segal: A note on the pseudo arc.

In his paper (Homogeneous Spaces, Duke Math. J. vol. 20 (1953) pp. 321-329) J. R. Isbell raised the following question: Is every shrinkable connected topological space locally connected? The purpose of this note is to answer the

question in the negative by showing the pseudo arc to be a shrinkable connected space which is not locally connected. (Received September 30, 1957.)

540-6. D. J. Foulis: Induced isomorphisms of tensor products. Preliminary Report.

Let H be a subgroup of the abelian group G, $j:H \rightarrow G$ the injection map. Let B be an abelian group, $i:B \rightarrow B$ the identity map. Denote the induced homomorphism $j \otimes i: H \otimes B \rightarrow G \otimes B$ by j'. In case H is a direct summand of G, it is well known that j' is an isomorphism into, and, that the image of $H \otimes B$ in $G \otimes B$ under j' is a direct summand of $G \otimes B$. An elementary proof is given to show that the words "direct summand" above may be replaced by the words "pure subgroup"; and that, conversely, if j' is an isomorphism into for every abelian group B, H is necessarily a pure subgroup of G. (Received September 30, 1957.)

540-7. H. C. Griffith: Tame surfaces in three-space.

The definition of property $\mathcal{P}$ for arcs given by Harrold (Duke Math. J. vol. 21 (1954) pp. 615-622) suggests the following condition on the imbedding of a 2-cell C in $R^3$. If p, A, and $\epsilon$ are, respectively, a point, a sub-arc of C on p, and a positive number, then there is a topological 2-sphere in $S(p,\epsilon)$ enclosing p, meeting C in a connected sub-set of a 1-sphere, and meeting T in a set with the order of p in T as cardinal. This property, together with the strong enclosure property (Griffith, Trans. Amer. Math. Soc., vol. 81 (1956) pp. 25-48) constitute a set of necessary and sufficient conditions in order that a 2-cell be tame in $R^3$. (Received September 30, 1957.)

540-8. R. D. Anderson: A certain two-dimensional continuum.

A characterization of a certain 2-dimensional locally connected continuum is given. The continuum M is a 2-dimensional analogue of the universal curve (not necessarily homeomorphic to other known analogues). The characterization employs a sequence of finite closed coverings of M, a finite sequential condition of peelableness somewhat like that employed by Bing in his characterization of 3-space (Trans. Amer. Math. Soc. vol. 70 (1951) pp. 15-27) and a condition like that of "interlacing" used by the author in his characterization of the universal curve (to appear in Annals of Mathematics). (Received September 30, 1957.)

540-9. A. T. Brauer and A. C. Mewborn: Intervals for the characteristic roots of an Hermitian matrix.

Recently Ky Fan [National Bureau of Standards Applied Mathematics Series. No. 39 (1954) pp. 131-139] obtained upper and lower bounds for each of the characteristic roots of an Hermitian matrix. In this paper another solution for the same problem is given which is often simpler than the method of Ky Fan. (Received September 30, 1957.)

540-10. R. P. Hunter: Compact connected topological semigroups.

Let S be a topological semigroup, (mob), which is a continuum irreducible between two points with $S^2 = S$ and proper minimal ideal. Several theorems of Faucett, [Proc. Amer. Math. Soc. vol. 6 (1955) pp. 748-756], dealing with the case in which S is irreducibly connected between two points are extended to this situation. Cut points are replaced by the continua T(p), the points at which S is not aposyndetic with respect to p. [Jones: Amer. J. Math. vol. 63 (1941) pp. 545-553]. Using the above notions a result of Koch and Wallace is extended as follows; The Rees quotient S/K is irreducibly connected between two points one of which is idempotent. If S has neither left nor right unit then both the above points are idempotent, K separates, and S/K is the sum of two abelian clans. If S is irreducible between two idempotents with no other idempotents and K not prime, then S is a calabi clan. If S is the essential sum of n indecomposable continua [Swingle: Amer. J. Math. vol. 52 (1930) pp. 647-658] then $S^2 = S$ implies S is either all left or all right zeros. (Received September 30, 1957.)

540-11. Nickolas Heerema: Note on complete discrete valuation rings.

The following theorem is proved. Let $R_1$ and $R_2$ be complete discrete valuation rings of characteristic zero with isomorphic residue fields of characteristic p and let $R_1$ be unramified. Then $R_2$ is isomorphic to $\overline{R_1[x]}/I$ where $\overline{R_1[x]}$ is the ring of power series in x over R and I is a principal ideal generated by $p - ux^n$ where u is a unit in $\overline{R_1[x]}$ and n is the ramification index of $R_2$. Conversely, any such ring $\overline{R_1[x]}/I$ is a complete discrete valuation ring with ramification index n and residue field isomorphic to that of $R_1$. (Received September 30, 1957.)

540-12. P. S. Mostert and A. L. Shields: One-parameter semigroups in a semigroup.

Let S be a topological semigroup with identity 1. A one-parameter semigroup is a function $f:[0,1] \to S$ such that $f(0) = 1$, and $f(a + b) = f(a)\, f(b)$ when $a, b, a + b \in [0,1]$. The authors prove the following: Theorem. Let S be a locally compact semigroup with identity 1 and H(1) denote the maximal subgroup containing 1. If H(1) contains a compact subgroup open in H(1), and if there exists a net $\{x_\alpha\}$, $x_\alpha \notin H(1)$, converging to 1 with the property that for some neighborhood V of 1, there exist positive integers $n(\alpha)$ such that $x_\alpha^{n(\alpha)} \notin V$, then there is a one-parameter subgroup f such that $f(a) \in H(1)$ if and only if $a = 0$. Corollary. If S is a compact semigroup with identity 1 which is not a cluster point for idempotents, and if H(1) is not open in S, then there is a one-parameter semigroup f such that $f(a) \in H(1)$ if and only if $a = 0$. (Received September 30, 1957.)

540-13. J. E. Maxfield and R. G. Selfridge: Lin's method applied to translated polynomials.

It has been shown that for f(x) a polynomial of degree $n \geq 5$ having real coefficients there exists a real number $T \geq 0$ such that for $t \geq T$ the Lin iterative method of extracting quadratic factors from a polynomial converges for the polynomial $f(x \pm t)$. (Received September 30, 1957.)

540-14. Trevor Evans and F. L. Hardy: Sheffer stroke functions in many-valued logics.

For a given n-valued propositional logic $\mathscr{L}$ does there exist a wff in two propositional variables, such that the associated function of this formula generates, under composition of functions exactly those functions on the set T of truth-values associated with formulas of $\mathscr{L}$? If this is so, then $\mathscr{L}$ can be defined in terms of a single binary connective. Such a connective is a generalization of the Sheffer Stroke function for 2-valued logic. For Post n-valued logics Sheffer Stroke functions have been obtained by Webb, Martin and others. A new class of Sheffer Stroke functions is obtained here and of the $n^{n^2}$ functions of two variables these comprise $n^{n^2 + 5n + 6} \cdot (n - 1)^{n-2}$. The corresponding problem for the n-valued Lukasiewicz-Tarski logics is complicated by the fact that these are not functionally complete. However, two Sheffer Stroke functions are obtained for the logics in case $n \not\equiv 1 \pmod 3$. (Received September 30, 1957.)

540-15. A. D. Wasel: On the variation of parameters.

The following theorem is proved. Given that in a domain $A, f_i(x_1,x_2,\ldots,x_n,\dot{x}_1,\dot{x}_2,\ldots,\dot{x}_n; a_1,a_2,\ldots,a_{2n}) = 0, i = 1,2,\ldots,2n$ (the dots indicating differentiation with respect to t and the $a_i$ being parameters) and the conditions of the Implicit Function Theorem are satisfied, then $\dot{a}_i = -(1/D)\sum_{j=1}^{n}[\partial(f_1,f_2,\ldots,f_{2n})/\partial(a_1,a_2,\ldots,a_{i-1},\dot{x}_j,a_{i+1},\ldots,a_{2n})]\delta\ddot{x}_j$, $i = 1,2,\ldots,2n$, where $\delta\ddot{x}_j$ are perturbations in $\ddot{x}$ corresponding to variations with t in $a_i$ and D is the Jacobian of the system. The equivalence of $a_i$ as given by the above expressions to those derived by "the method of the variation of parameters" is shown. (Received September 30, 1957.)

540-16. Tomlinson Fort: A problem of Richard Bellman.

In the January 1957 issue of the Bull. of Amer. Math. Soc. under the heading "Research", Richard Bellman proposes several questions relative to a Sturm-Liouville problem involving two parameters. In the present note all questions asked by Bellman are completely answered. The method employed is the method of differences. (Received October 1, 1957.)

540-17. Nosup Kwak: Embedding compact metric spaces in cubes.

The motivating question, still unanswered, is whether or not a compact metric space X of dimension k, which can be embedded in $I^n$ (the n-dimensional cube), can be embedded in $\mathcal{M}_n^k$, the subset of $I^n$ such that each point has at most k rational coordinates. For $k = n - 3$, the following statements are equivalent, where H is the space of homeomorphisms of X into $I^n$ with the usual metric topology: (1) The set of $f \in H$ such that $f(X) \subset \mathcal{M}_n^{n-3}$ is a dense $G_\delta$ subset of H; (2) if p is a projection of $I^n$ onto one of its faces, then the set of $f \in H$ such that $\dim p(f(X)) \leq n - 3$ is dense in H; (3) the set of $f \in H$ such that $m_{n-2} f(X) = 0$ (for definition, see Hurewicz-Wallman: Dimension theory, p. 102) is dense in H; and (4) the set $A \subset H$, defined below, is a dense $G_\delta$ subset of H: $f \in A$ if and only if (i) if P is any r-dimensional linear subset of $I^n$ $(n - 3 \leq r \leq n)$ then $\dim (f(X) \cap P) \leq r - 3$ and (ii) if $\pi$ is an $(n - 4)$-dimensional linear subset of $I^n$ obtained by fixing four coordinates at rationals, then $f(X) \cap \pi = \phi$. (Received October 1, 1957.)

540-18. F. M. Reza: On the Schlicht behaviour of certain impedance functions.

Theorem: Let $A_k > 0$ $(k=0,1,2,\ldots n)$ and $0 < a_1 < a_2 < \ldots < a_n$; then the function $W(z) = A_0 + \sum_{k=1}^{n} A_k/(z + a_k)$ is Schlicht in the right half plane. In the terminology of electrical network theory, the above theorem states that the driving-point impedance of a passive network composed of resistors and inductors or the driving-point admittance of a passive network composed of resistors and capacitors is a Schlicht function of the complex frequency in the right half plane. The theorem is further generalized to cover a larger sub-class of positive real functions. (Received October 2, 1957.)

540-19. Andrew Sobczyk: Pseudo-ergodic random processes. Preliminary Report.

If a random process $\{x(t)\}$ is stationary, the ensemble average $E\{x(t)\} = B(t)$ is a constant. The stationary process is ergodic if the time average $\lim_{T\to\infty} (1/2T)\int_{-T}^{T} x(t)dt = A(x)$ is equal to the same constant B, for almost all functions x(t) of the process. (For bounded functions x(t), the time average is independent of the choice of origin.) A process is pseudo-ergodic if the time average A(x) is a constant C for almost all functions x(t). The covariance of a process is $E\{x(t)x(t+\tau)\} = \phi(t,\tau)$. The correlation of a member function x(t) of the process is $\lim_{T\to\infty} (1/2T)\int_{-T}^{T} x(t)x(t+\tau)dt = A[x(t)x(t+\tau)] = R(\tau)$. In case of a stationary ergodic process, the correlation $R(\tau)$ for almost all functions x(t) is equal to the covariance $\phi(0,\tau)$ of the process. By use of the covariance and correlation, in this paper possible types of one- and two-dimensional pseudo-ergodic processes are determined, and some of their properties are derived. (Received October 2, 1957.)

540-20. G. B. Preston: An embedding theorem for semigroups.

We prove the following theorem: Any semigroup can be embedded in a regular $\mathcal{D}$-simple semigroup with identity. A semigroup S is said to be regular if $a \in aSa$ for all $a \in S$. A semigroup is said to be $\mathcal{D}$-simple if it consists of a single $\mathcal{D}$-class (Cf. D. D. Miller and A. H. Clifford, Regular $\mathcal{D}$-classes in semigroups, Trans. Amer. Math. Soc. vol. 82 (1956) pp. 270-280). If we suppose that the semigroup S contains an identity then S is $\mathcal{D}$-simple if and only if to each ordered pair of elements $a,b \in S$ there correspond elements $x,y,u,v \in S$ such that $ax = ub$, $axy = a$, $vub = b$. If S is any semigroup then the theorem is

proved by successively adjoining to S elements x,y,u,v which are solutions to the above equations and where a,b belong either to S or to a semigroup already constructed from S by this process. The construction can be terminated in a $\mathcal{D}$-simple semigroup in which S is embedded. (Received October 2, 1957.)

540-21. S. R. Struik: To the affine theory of area.

Hilbert's three concepts, utilized in his euclidean axiomatic theory of polygon areas (D.Hilbert, Grundlagen der Geometrie, 7. Aufl. 1930: zerlegungsgleich p. 70, ergänzungsgleich p. 70, von gleichem Inhaltsmass p. 78) of equal area, content, measure (translation by E. J. Townsend 2. ed., 1921, p. 58, 58, 65 resp.) based on the congruence axioms and the theory of proportions together with the algebra of segments, are checked against the analogous affine definitions and theory, omitting axioms irrelevant to a theory of area. A special case of the Cavalieri principle is made the basis which Hilbert uses implicitly in his congruence axioms of triangles. (Received October 2, 1957.)

540-22. M. L. Curtis and M. K. Fort, Jr: The fundamental group of one-dimensional spaces.

The authors have previously shown that for a separable metric connected one-dimensional space X, $\pi_k(X) = 0$ for all $k > 1$. It follows that $\pi_1(X)$ determines all of the singular homology groups of such a space. It is known that $\pi_1(X)$ need not be free, but it is shown in the present paper that (1) it has no elements of finite order and (2) the only finitely-generated abelian subgroups are cyclic. This suffices to show that the 2-dimensional singular group $H_2(X,Z)$ vanishes. If X is also compact and locally connected, then $\pi_1(X)$ is either finitely generated and free or it contains a certain specific uncountably generated (and nonfree) subgroup. (Received October 2, 1957.)

540-23. P. F. Conrad: The groups of word preserving permutations of a group.

Let F be a free group of countable rank with a set I of independent generators. For each $\prod_{i=1}^{n} x_i^{e_i}$ in F (where $x_i \in I$ and $e_i = \pm 1$), let $[\prod_{i=1}^{n} x_i^{e_i}]$ be the group of all permutations $\pi$ of G such that $(\prod_{i=1}^{n} y_i^{e_i})\pi = \prod_{i=1}^{n} (y_i\pi)^{e_i}$ for all sets $\{y_i\}_{i=1}^{n} \in G$ for which $y_i = y_j$ if $x_i = x_j$. The structure of the groups $[\prod x_i^{e_i}]$ and the relationship between G and these groups is investigated. $[x^n] = [x^{-1}]$ if and only if $g^{n+1} = e$ for all $g \in G$ or G is of order

3 and $3 \nmid n - 1$. G contains no elements of order 2,3,...,n if and only if $[X^{-k}] \subseteq [XX^{-1}]$ for $k = 1,2,\ldots,n - 1$. G is of bounded order if and only if $[XX^{-1}] \subseteq [X^k]$ for some integer $k \neq 0$ or 1. If $a^n = b^n$ implies $a = b$ for all $a,b \in G$, then $[X^n]$ is a normal splitting extension of A by B, where A is isomorphic to a direct product of cyclic groups, and B is isomorphic to a direct product of symmetric groups. $[XY^{-1}Z]$ is the holomorph of G, and G is abelian if and only if $[XY^{-1}Z] = [XYZ^{-1}]$. (Received October 2, 1957.)

540-24. Walter Feit: p-Extensions of p-adic number fields.

Let k be a p-adic number field of degree n over the p-adic rationals, let $d = 1$ if k does not contain the p-th roots of unity, $d = 2$ if otherwise. The following result is proved, generalizing a theorem of Šafarevič, (Amer. Math. Soc. Translations, Series 2, vol. 4, p. 59.) There is a one to one correspondence between normal extension of k whose degree is a power of p and normal subgroups N of the free group S on n + d generators whose index in S is a power of p, furthermore S/N is the Galic group of the extension which corresponds to N. (Received October 2, 1957.)

540-25. L. E. Ward, Jr: On dendritic sets.

A dendritic space is a connected and locally connected Hausdorff space in which each two points can be separated by some third point. A tree is a compact dendritic space. In somewhat the same spirit of the author's earlier papers (Proc. Amer. Math. Soc. vol. 5, pp. 992-994 and vol. 8, pp. 798-804; Pacific J. Math., to appear) dendritic spaces are characterized topologically and in terms of their inherent order properties. If a dendritic space is convex in the sense of Nachbin (Comptes Rendus, vol. 226, pp. 381-382) then it admits a compactification as a tree. (Received October 2, 1957.)

540-26. S. L. Warner: The topology of compact convergence on continuous function spaces.

Let T be a completely regular space, E the vector space of all continuous real-valued functions on T, equipped with the compact-open topology. We seek to relate properties of T with properties of locally convex space E; among other results, we cite the following: E is a Fréchet space if and only if T is a hemicompact k-space. E has a countable dense set if and only if the topology of T is stronger than a separable metrizable topology on T. E is symmetric (quasi-

tonnelé) if and only if for every closed noncompact subset S of T, there exists a real-valued lower semi-continuous function $x \geqq 0$ on T such that x is unbounded on S but bounded on every compact subset of T. E has a denumerable fundamental system of bound sets if and only if T is pseudo-compact and every Cauchy sequence in E converges. E is a (DF)-space if and only if the union of any countable family of compact subsets of T is relatively compact. In addition, we are able to settle a problem of Michael's concerning full locally m-convex algebras and one of Grothendieck's concerning Mackey's conditions of convergence. (Received October 2, 1957.)

540-27. F. B. Jones: R. L. Moore's Axiom 1' and metrization.

Suppose that a Hausdorff space S has a development $G_1, G_2, \ldots$ [see Bing, Canadian J. Math. vol. 3, p. 180] such that if H and K are nonintersecting closed sets one of which is compact there exists an integer n such that no element of $G_n$ intersects both of them. It is shown, using a metrization theorem due to R. L. Moore, that S is metric. The converse is evident. (Received October 2, 1957.)

540-28. F. B. Jones: A remark on n-point aposyndesis and embedding a cartesian product in the plane.

If M is the cartesian product of two nondegenerate Hausdorff continua and G is the collection of all finite subsets of M, it is shown that M is aposyndetic with respect to G (i.e., if $p \in M$ and $g \in G$ such that $g \subset M - p$, there exist a continuum H and an open subset U of M such that $p \in U \subset H \subset M - g$). It follows as a corollary, that if M is the cartesian product of two nondegenerate compact metric continua and M is embeddable in the plane (or $S^2$) then M is locally connected (in fact, M must be either a disk or an annulus). (Received October 2, 1957.)

540-29. P. M. Swingle: Connected sets of Van Vleck.

We extend the methods of Van Vleck [Trans. Amer. Math. Soc. vol. 9 (1908) pp. 237-244] to obtain in $E_m$, $m > 1$, Lebesgue nonmeasurable connected sets, which we call connected sets of Van Vleck. We show: $E_m = M + W + W_1 + W_2 + \ldots + W_i + \ldots + W_t$, where $t = 2^m - 1$, each $W_i$ is a reflection of W about some $(m - j)$-coordinate subspace $(j = 1, 2, \ldots, m - 1)$; W, and so each $W_i$, is a connected set of Van Vleck, and M is of measure zero. Here W can be any one

of the following: (1) an indecomposable connected set; or (2) if R is any region where $R \cdot W \neq 0$, $R \cdot W$ is a connected set of Van Vleck; or (3) if $m = 2$, W is a biconnected set with dispersion point. Also M, W, and the $W_i$ are disjoint sets, each dense in $E_m$. (Received October 3, 1957.)

540-30. Naoki Kimura and Miyuki Yamada: On some special idempotent semigroups (II).

An idempotent semigroup is called normal (or left normal) if $xyzw = xzyw$ (or $xyz = xzy$) is satisfied for any x, y, z, w. Then a normal idempotent semigroup is expressed as a special kind of product of a left normal idempotent semigroup and a right normal idempotent semigroup. The structure of left (right) normal idempotent semigroups is completely determined by the family of mappings, and thus that of normal idempotent semigroups is also completely determined. From this any idempotent semigroup with some identities of the forms: $x_1x_2\cdots x_n = x_{p_1}x_{p_2}\cdots x_{p_n}$, where $\begin{pmatrix} 1 & 2 & \cdots & n \\ p_1 & p_2 & \cdots & p_n \end{pmatrix}$ is a permutation, will be determined in detail. (Received October 3, 1957.)

540-31. Gert Sabidussi: On a class of fixed-point-free graphs.

Suppose a connected graph X has the following two properties. (1) The automorphism group G(X) of X is nontrivial. (2) If x and y are two distinct vertices of X then there is at most one $\phi \in G(X)$ such that $\phi x = y$. It is shown that under these conditions X is a generalized Cayley color-group of G(X) with respect to a set of generators of G(X). (Received October 4, 1957.)

540-32. H. L. Loeb: Rational fraction approximations at discrete points.

Let $a^1,\ldots,a^\nu$ be vectors in $E_n$ of rank n. Similarly $b^1,\ldots,b^\nu$ are vectors in $E_m$ of rank m; $\nu > m + n$. Let $\phi_1,\ldots,\phi_\nu$, $\delta_1, \delta_2$ be real numbers where $\delta_2 > \delta_1 > 0$, and define $S = \{x \in E_{n+m} : \delta_1 \leqq \sum_{\ell=1}^{m} x_{n+\ell} b_\ell^j \leqq \delta_2;\ j = 1,\ldots,\nu\}$. Then $x^0 \in S$ will be called a solution if and only if $\max_j |R_j(x^0)| \leqq \max_j |R_j(x)|$ for all $x \in S$ where $R_j(x) = \phi_j - \sum_{i=1}^{n} x_i a_i^j / \sum_{\ell=1}^{m} x_{n+\ell} b_\ell^j$ $(j = 1,\ldots,\nu)$. Questions concerning existence and uniqueness of a solution are investigated. For example let $x^0$, an interior point of S, be a solution. Theorem: If $|R_1(x^0)| = |R_2(x^0)| = \cdots = |R_p(x^0)| > |R_{p+j}(x^0)|$ $(j = 1,\ldots,\nu - p)$ where $p \geqq m + n$, and if for each set of $m + n - 1$ indices $(k_1,\ldots,k_{m+n-1})$ selected from among the first p indices the matrix $(\partial R_{k_j}(x^0)/\partial x_i)$ $(i = 1,\ldots,m + n)$ $(j = 1,\ldots,m + n - 1)$

has rank m + n - 1, then all solutions are scalar multiples of $x^0$. Sufficient conditions for uniqueness are also established when the solution is a boundary point of S. (Received October 11, 1957.)

## THE NOVEMBER MEETING IN COLUMBIA

November 29-30, 1957

541-1. D. F. Dawson: Continued fractions with absolutely convergent even or odd part.

Let f(a) denote the continued fraction $1/\overline{1 + a_1}/\overline{1 + a_2}/\overline{1 + \ldots}$ and g(b) denote the continued fraction $1/\overline{b_1 + 1}/\overline{b_2 + 1}/\overline{b_3 + \ldots}$, where each of a and b is a complex number sequence. Theorem. If the odd and even parts of g(b) converge, the odd (even) part absolutely, then g(b) converges if the series $\sum|b_{2p-1}$ (the series $\sum|b_{2p}|$) diverges. Theorem. If there exist a number sequence $\{r_{2p}\}_{p=0}^{\infty}$ and a number M such that $0 < r_{2p} \leq 1$, $p=0,1,2,\ldots$, $0 < M \leq \prod_{i=1}^{n} r_{2i-2}$, $n=1,2,3,\ldots$ and $r_{2p}|1 + a_{2p-1} + a_{2p}| \geq r_{2p}r_{2p-2}|a_{2p-1}| + |a_{2p}|$, $p=1,2,3,\ldots$, then f(a) converges if some $a_{2q} = 0$ and $a_{2p-1} \neq 0$, $p=1,2,3,\ldots$, or in case no term of a is zero and the series $\sum|b_{2p-1}|$ diverges. Theorem. If there exist a number sequence $\{r_{2p}\}_{p=0}^{\infty}$ and a number r such that $0 < r_{2p} \leq r < 1$, $p=0,1,2,\ldots$, and $r_{2p}|1 + a_{2p-1} + a_{2p}| \geq r_{2p}r_{2p-2}|a_{2p-1}| + |a_{2p}|$, $p=1,2,3,\ldots$, then f(a) converges absolutely if one of the sequences $\{a_{2p-1}\}_{p=1}^{\infty}$ and $\{a_{2p}\}_{p=1}^{\infty}$ contains no zero term. In connection with each of these theorems an example is given which shows the theorem to be "best" in a certain sense. (Received May 23, 1957.)

541-2. Samuel Kaplan: The second dual of the space of continuous functions, II.

We use the notation of the first paper (Trans. Amer. Math. Soc. vol. 86 (1957) pp. 70-90). The duality between L and M with respect to order-convergence (now generally recognized as more natural than topology for this duality) is first described. It is then shown that the smallest set in M which contains C and is closed under order-convergence is M itself. Next the weakly compact subsets of L and, dually, the Mackey topology $\tau(M,L)$ are characterized; the latter turns out to be given by the order in M in a natural way. The relations between Radon measures, regular measures, countably additive measures, and general finitely additive measures on M, U, Bo, and Ba are studied. Finally some relations between closed ideals, order-continuity, and the topology $|w|(\cdot,\cdot)$ (for general vector lattices) are given. (Received August 12, 1957.)

541-3. L. A. Kokoris: Nodal flexible associative-admissible algebras.

Nodal noncommutative Jordan algebras were defined and studied by R. D. Schafer (On noncommutative Jordan algebras, to appear in Proc. Amer. Math. Soc.). All known examples of such algebras are associative-admissible (see the author's Some nodal noncommutative Jordan algebras, to appear in Proc. Amer. Math. Soc.). Let A be a finite dimensional flexible algebra over a field F whose characteristic is not 2 and let $A^+$ be associative. Then A is nodal if $A = F1 + N$ where 1 is the identity element of A and N is the subspace of nilpotent elements of A. There exist elements $x_1,\ldots,x_n$ in N such that every element in N is a polynomial in $x_1,\ldots,x_n$ where the polynomial product is that of $N^+$. Multiplication in A is given by the formula $fg = f\cdot g + \sum_{i,j} \partial f/\partial x_i \cdot \partial g/\partial x_j \cdot(\alpha_{ij}1 - x_i\cdot x_j + w_{ij})$ where f, g are any polynomials in $1,x_1,\ldots,x_n$; $x_i x_j = \alpha_{ij}1 + w_{ij}$, and the dot product is that of $A^+$. The algebra A has characteristic $p \neq 2$. It is simple if and only if every $x_i^p = 0$ and A has dimension $p^n$. (Received August 23, 1957.)

541-4. W. R. Abel and L. M. Blumenthal. Distance geometry of metric arcs. II.

A metric continuum K has property $T_{\pi/2}$ ($T^*_{\pi/2}$) at a point p of K if and only if there exists $\delta_p>0$ (and $\alpha_p>0$) such that each triple of pairwise distinct points of $K\cdot U(p;\delta_p)$ contains an angle greater than or equal to $\pi/2$ (greater than or equal to $\pi/2 + \alpha_p$). The principal result proves that a metric continuum with property $T^*_{\pi/2}$ at a point p is a rectifiable arc in a neighborhood of p. The well-known theorem of Pauc concerning metric continuum with finite Menger curvature at a point follows as a corollary. For metric arcs with property $T_{\pi/2}$ at each point, almost all n-lattices are unique and are homogeneous chains. Some useful properties of the class $\Gamma_\beta$ of arcs arising from the unit segment [0,1] by defining $\text{dist}(x,y) = [\text{euclid.dist}(x,y)]^\beta$ $0<\beta<1$ are derived. (Received September 19, 1957.)

541-5. D. O. Ellis: Remarks on the elementary symmetric functions.

Certain properties are elaborated for the arithmetic analogue of the join function in a Boolean ring. Also examined is the significance of the lattice-theoretic analogues of the elementary symmetric functions. The methods employed are elementary. (Received September 30, 1957.)

541-6. W. S. Loud: Periodic solutions of $x'' + cx' + g(x) = \epsilon e(t)$. II.

In what follows the notation of Bull. Amer. Math. Soc. Abstract 62-4-500 is used. The case is studied for which all hypotheses of the above abstract are satisfied except that e(t) is orthogonal to both $x_0'(t)$ and $x_0''(t)$. Using implicit function methods the following existence and stability results for periodic solutions of (1) $x'' + cx' + g(x) = \epsilon e(t)$ are obtained. Let $h_1(t)$ be that periodic solution of $y'' + g'(x_0(t))y = e(t)$ with $h_1'(0) = 0$. Let $h_2(t)$ and $k_2(t)$ be the corresponding solutions of $y'' + g'(x_0(t))y = -g''(x_0(t))h_1(t)^2/2$, and $y'' + g'(x_0(t))y = -g''(x_0(t))x_0'(t)h_1(t)$. Then if e(t) is orthogonal to $h_1'(t)$ and if the equation in m, (2) $F(m) \equiv m^2 \int_0^L x_0''(t)e(t)dt + 2m \int_0^L k_2'(t)e(t)dt + 2\int_0^L h_2'(t)e(t)dt = 0$ has a positive discriminant, there exist one or two periodic solutions of (1) near to $x_0(t)$ according as (2) is linear or quadratic. If $m_0$ is a simple real root of (2) the corresponding periodic solution is $x_0(t) + \epsilon [h_1(t) + m_0 x_0'(t)] + o(\epsilon)$. The stability is determined by the algebraic sign of $F'(m_0)$ and the hardening or softening character of g(x). See also Kač, Prikl. Mat. i Meh. vol. 19 (1955) pp. 13-32. The research for this paper was supported in part by the Office of Ordnance Research. (Received September 30, 1957.)

541-7. J. H. DeHardt: A theorem on the convergence of empiric measures.

Let $\mu$ be a probability measure on the Borel sets in k-dimensional Euclidean space $E_k$. Let $\{X_n\}$ be a sequence of k-dimensional independent random vectors, distributed according to $\mu$. Define $\mu_n(A)$ to be the proportion of observations among $X_1,\ldots,X_n$ which fall in A. Let $\mathcal{O}_1$ be the class of Borel sets in $E_k$ such that each A in $\mathcal{O}_1$ has the following property: if $x = (x_1, x_2, \ldots, x_k)$ is in A and $y = (y_1, y_2, \ldots, y_k)$ is such that $y_i \leqq x_i$, for $i = 1,2,\ldots,k$, then y is in A. Let $\mathcal{O}_j$, $j = 2,\ldots,2^k$ be the $2^k - 1$ classes of sets obtained by reversing, one at a time, the k inequalities occurring in the definition of $\mathcal{O}_1$. Let $\mathcal{O} = \bigcup_{j=1}^{2^k} \mathcal{O}_j$. J. R. Blum (Annals of Math. Stat., Sept., 1955) proves that if $\mu$ is absolutely continuous with respect to Lebesgue measure then, $P[\lim_{n\to\infty} \sup_{A\in\mathcal{O}} |\mu_n(A) - \mu(A)| = 0] = 1$, and conjectures the conclusion follows for arbitrary $\mu$. This paper shows that the following condition on $\mu$ is necessary and sufficient for the conclusion: if A is the point set of any strictly monotone graph in $E_k$ and $A_1$ is the set of point masses of $\mu$, then $\mu(A - A_1) = 0$. (Received October 1, 1957.)

541-8. F. M. Wright: Some extensions of results for C-fractions of a set $\Gamma$.

Two theorems are proved which generalize results due to W. T. Scott (Ann. of Math. vol. 51 (1950) pp. 56-57), and discussed from another point of view by the author (Bull. Amer. Soc. vol. 62 (1956) p. 585), relative to the set $\Gamma$ of all C-fractions C(w) with the property that the corresponding power series $Q(w) \equiv \sum_{n=0}^{\infty} \mu_n w^{n+1}$ is such that $P(z) \equiv Q(1/z)$ has a J-fraction expansion. Two additional theorems are proved which have significance for the set of absolutely regular elements of $\Gamma$, and which generalize some results of the author in the above paper which relate to these C-fractions. Some further determinant formulas for the coefficients of elements C(w) of the set $\Gamma$ are obtained. (Received October 1, 1957.)

541-9. H. D. Brunk: Uniform consistency of estimators of linearly ordered parameters.

Further results are obtained relating to the following problem of estimation. The nondecreasing function $\theta(t)$ is to be estimated, given sample values of random samples of specified sizes from populations with means $\theta(t_i)$, $i = 1,2,\ldots, k$. The estimators of the means $\theta(t_i)$ which are maximum likelihood estimators when the populations belong to a common exponential family were shown (Ayer, Brunk, Ewing, Reid, Silverman, An empirical distribution function for sampling with incomplete information, Ann. Math. Stat. vol. 26 (1955) pp. 641-647; Brunk, Maximum likelihood estimates of monotone parameters, loc. cit., pp. 607-616) to be (weakly) consistent. Conditions are now obtained for strong uniform consistency over an interval, using a geometric interpretation of the estimators due to W. T. Reid and a slightly strengthened form of the strong law of large numbers. An asymptotic lower bound is also obtained for the probability of achieving a prescribed uniform precision over an interval. (Received October 2, 1957.)

541-10. D. E. Edmondson: A modular topological lattice.

A compact, connected two dimensional portion of the closed three cell, which cannot be embedded in the plane, is given an order in such a way that it is a modular nondistributive topological lattice. The example is used to show the arithmetic structure of the lattice. The example further serves as the foundation of a generalization which gives an example of a compact, connected topological lattice without a basis of open sublattices. (Received October 2, 1957.)

541-11. R. P. Gosselin: On the divergence of Fourier series.

Let $s_n(x;f)$ be the nth partial sum of the Fourier series of f. Given any sequence $\{n_\nu\}$ of positive integers tending to $\infty$, there is an integrable function f such that $s_{n_\nu}(x;f)$ diverges for almost every x. The construction is carried out by modification of Kolmogoroff's well known example (cf. A. Zygmund, Trigonometrical series, Warsaw, 1935, p. 175). (Received October 2, 1957.)

541-12. L. M. Kelly: Metric curvature in linear spaces.

In general normed linear spaces there are arcs with vanishing Menger curvature at each point which are not metric segments (see I. J. Schoenberg Proc. Amer. Math. Soc. vol. 3 (1952) pp. 961-964). This note is concerned with the conjecture that this phenomenon cannot occur in uniformly convex spaces. Partial results in finite dimensional spaces are obtained and explicit formulas for calculating various metric curvatures are developed. (Received October 2, 1957.)

541-13. Eugenio Calabi: Improper affine hyperspheres of convex type and a generalization of a theorem by K. Jörgens.

An application of the maximum principle for functions satisfying elliptic inequalities leads to the following Theorem I. If $V_n$ is a locally convex, improper affine hypersphere in the affine (n+1)-space $E_{n+1}$, p any point of $V_n$, $J = J(p)$ the Pick invariant (Blaschke, Vorlesungen über Differentialgeometrie, vol. II) and $\gamma(p)$ the geodesic distance of p from the complement of $V_n$ in its completion with respect to the Schwarz-Pick metric: then $J(p) < 3 \cdot 5/(\gamma(p))$. In particular, if $V_n$ is complete with respect to the metric, then it is a convex paraboloid. When $n \leqq 4$ Theorem I implies the following result in partial differential equations: Theorem II. Let u be a locally convex function of n variables $(x^i)$, satisfying the equation $\det.(\partial^2 u/\partial x^i \partial x^j) = 1$ in a domain D including the origin 0. If u is of class $C^5$, if $n \leqq 4$, if $(\partial^2 u/\partial x^i \partial x^j)(0) = \delta_{ij}$, and if D contains the open ball with radius a and center 0, then $\sum_{i,j,k} [(\partial^3 u/\partial x^i \partial x^j \partial x^k)(0)]^2 \leqq C_n / a^2$ for a certain constant $C_n$. In particular, if D is the whole numerical n-space, then u is a quadratic polynomial in $(x^i)$. (Received October 3, 1957.)

## FEBRUARY MEETING IN NEW YORK

February 22, 1958

Note: The following group of abstracts is comprised of the abstracts which are listed on the program as being presented in person at the meeting. The abstracts of the papers presented by title and of late papers will be published in later issues of the NOTICES.

543-1. Withdrawn.

543-2. G. O. Losey: Ideals of index 1 in group rings.

Let $\mathcal{A}$ be a ring admitting as operators the elements of K, a commutative ring with unit. We shall say that an ideal $\Lambda$ of $\mathcal{A}$ has index 1 if $\mathcal{A}/\Lambda \cong K$. If $\Gamma$ is the group ring of a group G over K then any two ideals of index 1 in $\Gamma$ are isomorphic; moreover there is an automorphism of $\Gamma$ which carries one onto the other. The set of ideals of index 1 in $\Gamma$ can be put in one-one correspondence with the set of homomorphisms of G into the group of units of K, i.e. with the set of one dimensional representations of G in K. (Received November 27, 1957.)

543-3. Withdrawn.

543-4. L. J. Heider: A special class of Boolean algebras.

Let $\alpha$ denote a maximal dual ideal of Boolean algebra $\mathcal{B}$. Regard the Stone representation space $S(\mathcal{B})$ of $\mathcal{B}$ as the collection of all such ideals. Let $S_p(\mathcal{B})$ be the subspace of $S(\mathcal{B})$ consisting of all ideals $\alpha$ with the following property: for each sequence $\{a_n\}$ of elements of $\mathcal{B}$ in ideal $\alpha$ there is an element $a_0$ of $\mathcal{B}$ likewise in $\mathcal{B}$ with $a_0 \leqq a_n$ for all n. Finally, call $\mathcal{B}$ a P-Boolean algebra if the two following conditions are satisfied: (i) every element of $\mathcal{B}$ is in at least one ideal in $S_p(\mathcal{B})$; (ii) every collection $a_\alpha$, $\alpha \in \mathcal{A}$, of elements of $\mathcal{B}$ such that each ideal in $S_p(\mathcal{B})$ either contains some $a_\alpha$ or contains an element of $\mathcal{B}$ disjoint from all $a_\alpha$ has a least upper bound in $\mathcal{B}$. Then the subspaces $S^*$ with $v(S^*) = S_p(\mathcal{B})$ of spaces $S_p(\mathcal{B})$ found in the representation spaces $S(\mathcal{B})$ of P-Boolean algebras are exactly the P-spaces, i.e., the completely regular spaces X such that every prime ideal of the ring C(X) is a maximal ideal. A similar relation in a special class of Boolean algebras can be made for U-spaces, i.e., for completely regular spaces X such that with each element f of C(X) there is associated a unit element u of C(X) such that $f = u \cdot |f|$. (Received December 18, 1957.)

543-5. A. E. Nussbaum: Integral representation of semi-groups of unbounded self-adjoint operators.

Let $\mathcal{G}$ be a locally compact semi-group which can be imbedded in a locally compact group $\mathcal{G}$ and suppose that every bounded open set in $\mathcal{G}$ is of positive left or right Haar measure. Let $\{T_x; x \in \mathcal{G}\}$ be a weakly continuous semi-group of unbounded self-adjoint operators $T_x$ over $\mathcal{G}$ acting in a Hilbert space $\mathcal{H}$. That is, $T_{xy} \subseteq T_x T_y$ for all $x,y$ in $\mathcal{G}$ and $(T_x u,v)$ is continuous in $x$ for all $v$ in $\mathcal{H}$ and all $u$ in the common domain of the operators $T_x$. Let $\hat{\mathcal{G}}$ be the semi-group (under pointwise multiplication) of continuous homomorphisms $\chi$ of $\mathcal{G}$ into the real numbers topologized by the topology of uniform convergence on compact sets of $\mathcal{G}$. Suppose furthermore that there exists a denumerable set $D$ in $\mathcal{G}$ such that $x^{-1}D \cap \mathcal{G}$ or $Dx^{-1} \cap \mathcal{G}$ is not empty for every $x$ in $\mathcal{G}$. Then there exists a spectral measure $E(\sigma)$ relative to the Borel subsets $\sigma$ of $\hat{\mathcal{G}}$ such that $T_x = \int_{\hat{\mathcal{G}}} \chi(x) E(d\chi)$, where the integral converges in the strong operator topology. The result is obtained by imbedding the semi-group of self-adjoint operators in an algebra of unbounded operators using a theorem concerning the permutability of unbounded self-adjoint operators proved by the author in collaboration with A. Devinatz and J. von Neumann (Ann. of Math. vol. 62 (1955) pp. 199-203). (Received December 23, 1957.)

543-6. A. N. Feldzamen: Similarity and unitary equivalence for normal operators.

Two (bounded) normal operators that are similar are unitarily equivalent. That is, $N_1$, $N_2$ bounded and normal, and $T$, $T^{-1}$ bounded with $N_1 = TN_2T^{-1}$ implies $N_1 = UN_2U^*$, $U$ unitary. $U$ may be chosen as $T(T^*T)^{-1/2}$, and the result is valid for simultaneously similar sets of normal operators. The proof follows from the observation that $TE_2(\cdot)T^{-1}$ (where $E_i$ is the self-adjoint resolution of the identity for $N_i$, $i = 1,2$) is a resolution of the identity for $N_1$ in the sense of Dunford (Pacific J. Math. vol. 4 (1954) pp. 321-354) and therefore unique. Then $E_1(\cdot) = TE_2(\cdot)T^{-1}$, and the classical argument for the self-adjoint case shows $E_1(\cdot) = UE_2(\cdot)U^*$, $U$ being independent of the set variable. The result then suggests that the customary criterion for spatial indistinguishability in a multiplicity theory, unitary equivalence, be replaced by similarity. This replacement permits the immediate extension of previous multiplicity theories for normal operators to a family of non-normal operators on Hilbert space, the spectral operators of scalar type (Dunford, loc. cit.). (Received December 24, 1957.)

543-7. Daniel Shanks: A circular parity switch and applications to number theory.

A circular parity switch has a circular stator with 2n equally spaced divisions. At n of these there are contacts. Their locations are arbitrary except that no two contacts lie on a diameter. There is a rotor which may assume any of the 2n angular positions, and attached rigidly to it in any of the 2n positions are n hands. Their locations are again arbitrary except that no two hands lie in the same diameter. Theorem: As the rotor is turned the number of hands touching contacts is alternatingly even and odd. With this as a tool a short and most unusual proof is obtained for a generalization of that Gauss Lemma used in proving the reciprocity law. Theorem: If n is an odd prime the number of distinct rotors is $1 + (2^{n-1}-1)/n$. Thus a special case of the Fermat little theorem is an unexpected corollary. (Received January 6, 1958.)

543-8. Wilhelm Klingenberg: A characterization of Hermitian manifolds.

Let $E_G$ be a principal G-bundle over an (almost-) complex manifold $M^{2n}$, which is a reduction of the natural $GL_n(C)$-bundle over $M^{2n}$, G being a subgroup of $GL_n(C)$. If e.g. $G = U_n$, the unitary group $E_G$ is equivalent with having a Hermitian metric on $M^{2n}$. For $G = U_n$ there exists exactly one connection in $E_G$ with the property (P): The torsion form of the connection has no term of type (1,1). The same holds, if G is one of the open real forms of $GL_n(C)$. It will be proved, that this characterizes the real forms of $GL_n(C)$: Theorem 1: Let $E_G$ be a G-bundle over an (almost-) complex manifold $M^{2n}$. If and only if G is a real form of $GL_n(C)$, there exists exactly one connection in $E_G$ with property (P). The existence of an $E_G$ with G an open form of $GL_n(C)$ has topological implications on $M^{2n}$. $M^{2n}$ will be called general, if it has not one of these topological properties. Thus one gets a characterization of (almost-) Hermitian manifolds: Theorem 2: Let $E_G$ be a G-bundle over a general (almost-) complex manifold $M^{2n}$. If and only if $G = U_n$ there exists exactly one connection in $E_G$ with property (P). This is a counterpart of a characterization of Riemannian manifolds, due to H. Weyl and E. Cartan: Among the G-bundles over a real manifold $M^n$, G being a subgroup of $GL_n(R)$, precisely those with G an orthogonal group (with arbitrary signature $\neq 0$) are the ones, for which there exists exactly one connection with torsion form equal to zero. (Received January 7, 1958.)

543-9. John Wermer: Maximum principle for bounded functions.

Let F be an open Riemann surface and S a region on F bounded by a simple closed analytic curve $\Gamma$. Denote by A(S) the space of all functions bounded and analytic on $S \cup \Gamma$. Assume A(S) separates points on $S \cup \Gamma$ and assume the maximum principle for A(S), i.e. $|f(x)| \leqq \max_{t \in \Gamma} |f(t)|$ for all x in S and f in A(S). Assume also that for each t in $\Gamma$ there is some $Q_t$ in A(S) with $dQ_t \neq 0$ at t. Theorem: Under these hypotheses, S can be embedded in a finite Riemann surface S* such that for each f in A(S), if f* is the induced function on the image of S in S*, then f* has a bounded analytic extension to all of S*. In particular, S has finite genus. (Received January 8, 1958.)

543-10. Robert Heyneman: Duality in general ergodic theory.

Let G be a uniformly bounded semi-group of operators on a Banach space E. The orbit of a vector in E (resp.E*) is the closed convex hull of the trajectory of the vector under the action of G (resp. G*). Then if every orbit of E (resp. E*) contains at least one fixed point, any orbit of E* (resp. E) contains at most one fixed point. (G,E) is called ergodic if the vectors of E whose orbits contain 0, form a linear subspace. This condition amounts to a kind of strong uniqueness requirement on fixed points in orbits of E, and is implied by the existence of fixed points in every orbit of E*. In the reverse direction, if (G,E) is ergodic (resp. (G*,E*) is ergodic), then every weak -* compact orbit of E* (resp. every weakly compact orbit of E) contains at least one fixed point. In particular if E is reflexive, then (G,E) is ergodic if and only if every orbit of E* contains at least one fixed point under G*. These theorems throw light on many questions in general ergodic theory and a number of previous results become unified as corollaries. Applications to the subject of invariant means are given. (Received January 9, 1958.)

543-11. Ellen Correl: On a class of topologies for division rings. Preliminary report.

A topology t defined on a division ring D is said to be l.c. if the completion $(\hat{D},\hat{t})$ of (D,t) in the additive uniform structure is a locally compact, topological division ring. Theorem 1. If t is l.c. on D, and if (D,s) is a topological division ring such that s is not stronger than t, then the t-closure (s-closure) of each nonempty, s-open (t-open) subset of D is all of D. This theorem is used to prove a variety of results including the following: Theorem 2. If (D,t)

is a topological division ring, and if t is weaker than the join of l.c. topologies on D, then t is a join of l.c. topologies. Theorem 3. If, for each $\alpha \in A$ and $\beta \in B$, the topologies $t_\alpha$ and $t_\beta$ are l.c. on D, then $\bigvee_{\alpha \in A} t_\alpha = \bigvee_{\beta \in B} t_\beta$ if and only if A = B. (Received January 9, 1958.)

543-12. J. R. Büchi, C. C. Elgot and J. B. Wright: The nonexistence of certain algorithms of finite automata theory. Preliminary report.

Given classes $\mathcal{A}$ and $\mathcal{B}$ of formulas F[u] where u denotes a finite sequence of monadic predicates of natural numbers. Algorithms: (1) Solution. Given $F \in \mathcal{A}$ and $G \in \mathcal{B}$, to decide whether $F \rightarrow G$. (2) Solvability. Given $G \in \mathcal{B}$, to decide whether $\exists F \in \mathcal{A}$ such that $F \rightarrow G$. (3) Synthesis. Given $G \in \mathcal{B}$, to construct an $F \in \mathcal{A}$ such that $F \rightarrow G$, if such an F exists. Let $\mathcal{A}_0$ be the set of formulas $(\exists s)R[i,s,o]$ with input, auxiliary and output predicates i,s,o, where R characterizes an automaton (Burks and Wright, Proc. IRE vol. 41 (1953) pp. 1357-1365, Church, J. Symbolic Logic vol. 20 (1955) pp. 286-287). Let $\mathcal{B}_0$ be the set of formulas in i,o natural number variables, zero, successor and propositional connectives. Friedman (J. Symbolic Logic vol. 21 (1956) p. 219) and Alonzo Church (unpublished) include algorithms (1), (2), (3) for $\langle \mathcal{A}_0, \mathcal{B}_0 \rangle$. In contrast to these positive results, this paper proves the nonexistence of algorithms (1), (2), (3) for $\langle \mathcal{A}_0, \mathcal{B}_0 (\exists, +, \cdot) \rangle$. The proof is based upon the Gödel incompleteness theorem. The nonexistence of a combined solvability synthesis algorithm for $\langle \mathcal{A}_0, \mathcal{B}_0(\exists, +) \rangle$ is established utilizing Putnam (J. Symbolic Logic vol. 22 (1957) pp.39-54). For automata theory $(\exists s)R[i,s,o] \rightarrow G[i,o]$ means that the input-output transformation effected by the automaton R satisfies the condition G[i,o]. (Received January 9, 1957.)

543-13. Philip Hartman and Richard Sacksteder: On maximum principles for nonhyperbolic partial differential operators.

Let $z = z(x,y)$ be of class $C^2$, satisfy $Lz \equiv Az_{xx} + 2Bz_{xy} + Cz_{yy} + Dz_x + Ez_y \geqq 0$ on a bounded domain T and be continuous on T + T', where T' is the boundary of T. Let (H) denote the set of conditions: (i) A, B, C, D, and E are continuous on T, (ii) $AC \geqq B^2$, (iii) $A^2 + C^2 > 0$, and (iv) $A \geqq 0$. A vector $(x',y') \neq (0,0)$ is said to be in a characteristic direction at a point (x,y) of T if $C(x,y)x'^2 - 2B(x,y)x'y' + A(x,y)y'^2 = 0$, and a Jordan arc is said to be a characteristic arc if it has a continuously turning tangent which is in a characteristic direction at every point. It is shown that if (H) holds, $z = m (\leqq \max z)$

at some point $(x_0,y_0)$ of T, and $z \not\equiv m$ in every vicinity of $(x_0,y_0)$, then there is a characteristic arc through $(x_0,y_0)$ on which $z = m$. This local theorem is used to show that, under various additional hypotheses (e.g., if T is simply connected and if $(A,B,C) \neq$ const. (1,0,1) at every point of T), L has a weak maximum principle on T; i.e., $z = m$ at a point of T . In particular, (H) implies that a weak maximum principle holds locally in the sense that every point of T has a neighborhood on which L has a weak maximum principle. Related questions concerning the behavior of z at a point of T' when $z = m$, the validity of a strong maximum principle, and the possibility of lightening the condition $A^2 + C^2 > 0$ in (H) are considered. Some of the techniques used in the proofs are extensions of methods employed previously by E. Hopf and L. Nirenberg.

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